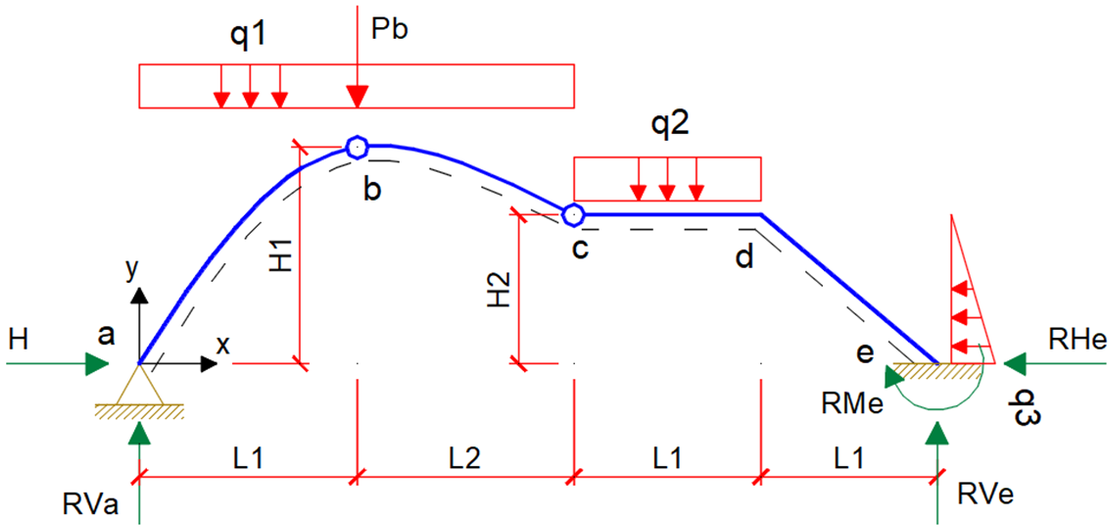
B2T4MA0622A

- Analisando o modelo estrutural abaixo, responda os questionamentos.



- Dados:

$L1 := 4,0\ m$ $L2 := 5,0\ m$ $H1 := 6,0\ m$ $H2 := 3,0\ m$

$q1 := 20\ kN/m$ $q2 := 15\ kN/m$ $q3 := 30\ kN/m$ $Pb := 40\ kN$

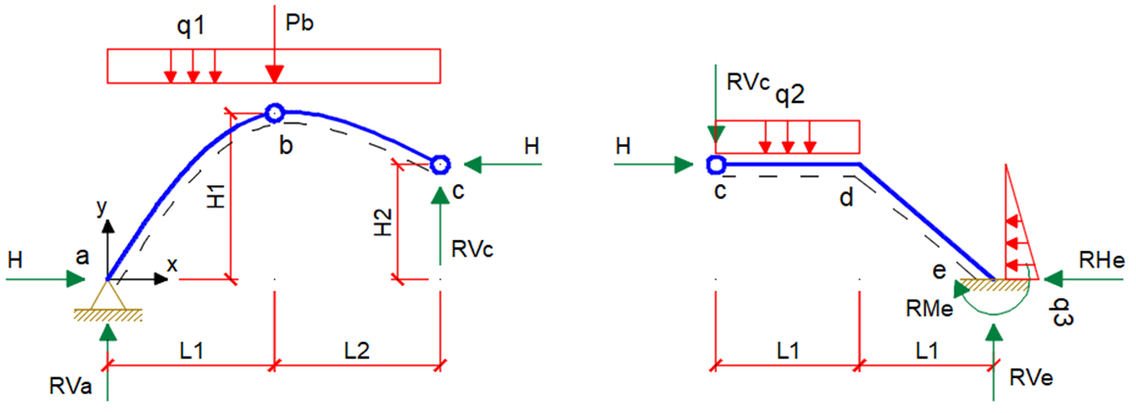
- Todas as barras são de seção transversal retangular de base "b", altura "h" e Módulo de Elasticidade "E":

$b := 0,20\ m$ $h := 0,80\ m$ $E := 50 \cdot 10^{6}\ kN/m^2$

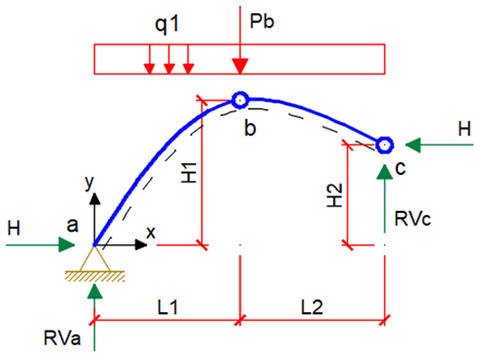
- O arco "abc" é parabólico de 2° grau.

Q01 - Qual o valor do momento fletor no arco "abc" na coordenada x = 3,0 m [em kNm e sinal conforme a convenção] ?
Q02 - Qual o valor do esforço cortante no arco "abc" na coordenada x = 4,0 m [em kN e sinal conforme a convenção] ?
Q03 - Qual o valor do esforço axial no arco "abc" na coordenada x = 7,0 m [em kN e sinal conforme a convenção] ?
Q04 - Qual o valor do esforço cortante no meio da barra "de" [em kN e sinal conforme a convenção] ?
Q05 - Qual o valor do esforço axial no meio da barra "de" [em kN e sinal conforme a convenção] ?
Q06 - Considerando somente efeitos de momentos fletores, qual o valor do deslocamento vertical do ponto "c" [em "mm" e com positivo para cima] ?

Decomposição da Estrutura:



Equacionamento da Geometria do Arco "abc" :



$xa := 0$ $\quad ya := 0$

$xb := L1 = 4$ $\quad yb := H1 = 6$

$xc := L1 + L2 = 9$ $\quad yc := H2 = 3$

$$matrizA := \begin{bmatrix} 1 & xa & xa^2 \\ 1 & xb & xb^2 \\ 1 & xc & xc^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 4 & 16 \\ 1 & 9 & 81 \end{bmatrix} \qquad vetorB := \begin{bmatrix} ya \\ yb \\ yc \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 3 \end{bmatrix}$$

$$vetorX := matrizA^{-1} \cdot vetorB = \begin{bmatrix} 0 \\ 2,4333 \\ -0,2333 \end{bmatrix}$$

$A := vetorX_1 = 0$

$B := vetorX_2 = 2,4333$

$C := vetorX_3 = -0,2333$

$$y(x) := A + B \cdot x + C \cdot x^2 = 0 + \frac{73}{30} \cdot x + \left(-\frac{7}{30}\right) \cdot x^2$$

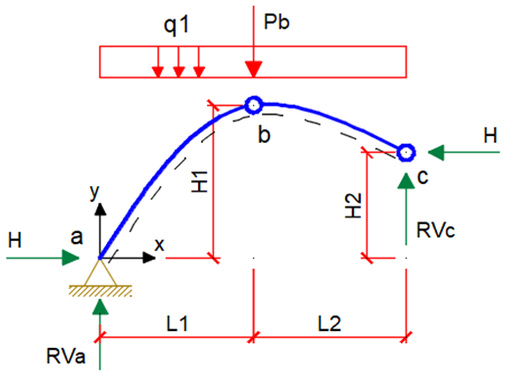
Reações de Apoio do Arco "abc" :

ΣMc = 0 (+) horário - olhando para "a"

```
RVa.(L1+L2) - H.H2 - Pb.L2 - q1.(L1+L2)^2 / 2 = 0
```

ΣMb = 0 (+) horário - olhando para "a"

```
RVa.L1 - H.H1 - q1.L1^2 / 2 =  0
```



Sistema de equações:

```
RVa.(L1+L2) - H.H2 = Pb.L2 + q1.(L1+L2)^2 / 2
RVa.L1 - H.H1 = q1.L1^2 / 2
```

$$matrizA := \begin{bmatrix} L1 + L2 & -H2 \\ L1 & -H1 \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ 4 & -6 \end{bmatrix} \qquad vetorB := \begin{bmatrix} \frac{q1 \cdot (L1 + L2)^2}{2} + Pb \cdot L2 \\ \frac{q1 \cdot L1^2}{2} \end{bmatrix} = \begin{bmatrix} 1010 \\ 160 \end{bmatrix}$$

$$vetorX := matrizA^{-1} \cdot vetorB = \begin{bmatrix} 132,86 \\ 61,905 \end{bmatrix}$$

$RVa := vetorX_1 = 132,86$

$H := vetorX_2 = 61,905$

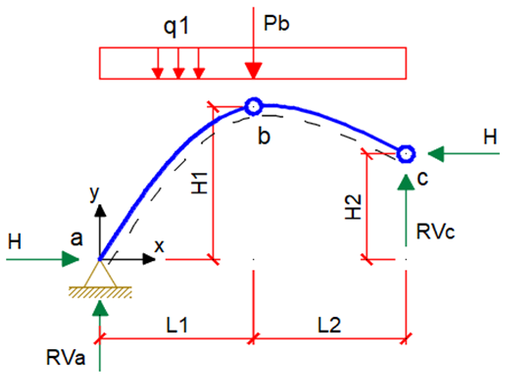
ΣFh = 0 (+) para direita: H igual em módulo porém com sentido contrário.

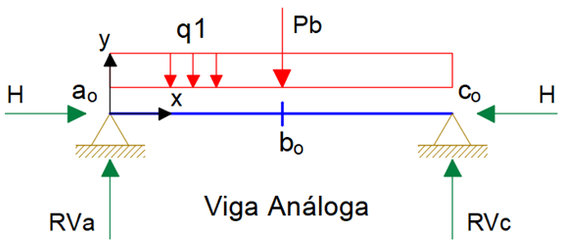
ΣFv = 0 (+) para cima

```
RVa + RVc - q1.(L1+L2)- Pb = 0
```

$RVc := -RVa + q1 \cdot (L1 + L2) + Pb = 87,143$

Equacionamento dos Esforços Internos no Arco "abc":





Equações de Momentos Fletores da Viga Análoga:

Trecho ab: $Mabo(x) := RVa \cdot x - \frac{q1 \cdot x^2}{2} = \frac{930}{7} \cdot x - \frac{20 \cdot x^2}{2}$

Trecho bc: $Mbco(x) := RVa \cdot x - \frac{q1 \cdot x^2}{2} - Pb \cdot (x - L1) = \frac{930}{7} \cdot x - \frac{20 \cdot x^2}{2} - 40 \cdot (x - 4)$

Equação de Esforços Cortantes da Viga Análoga:

Trecho ab: $Vabo(x) := \frac{d}{dx} Mabo(x) = -\frac{10 \cdot (-93 + 14 \cdot x)}{7}$

Trecho bc: $Vbco(x) := \frac{d}{dx} Mbco(x) = -\frac{10 \cdot (-65 + 14 \cdot x)}{7}$

Equação de Momentos Fletores do Arco:

$Mab(x) := Mabo(x) - H \cdot y(x)$

$Mab(x) = \frac{930}{7} \cdot x - \frac{20 \cdot x^2}{2} - \frac{1300}{21} \cdot \left(0 + \frac{73}{30} \cdot x + \left(-\frac{7}{30}\right) \cdot x^2\right)$ $\qquad Mab(3) = -13,333$

$Mbc(x) := Mbco(x) - H \cdot y(x)$

$Mbc(x) = \frac{930}{7} \cdot x - \frac{20 \cdot x^2}{2} - 40 \cdot (x - 4) - \frac{1300}{21} \cdot \left(0 + \frac{73}{30} \cdot x + \left(-\frac{7}{30}\right) \cdot x^2\right)$

$$V_s(x) = V_{so}(x).cos(\alpha(x)) - H.sen(\alpha(x))$$
$$N_s(x) = -V_{so}(x).sen(\alpha(x)) - H.cos(\alpha(x))$$

$$\alpha(x) = arctg\left(\frac{dy(x)}{dx}\right)$$

$\alpha(x) := \operatorname{atan}\left(\frac{d}{dx} y(x)\right) = -\operatorname{atan}\left(\frac{-73 + 14 \cdot x}{30}\right)$

Equações de Esforços Cortantes do Arco:

$Vab(x) := Vabo(x) \cdot \cos(\alpha(x)) - H \cdot \sin(\alpha(x))$

$Vab(x) = \left(-\frac{70 \cdot (-93 + 7 \cdot x + 7 \cdot x)}{49}\right) \cdot \cos\left(\operatorname{atan}\left(-\frac{27000 \cdot (-73 + 7 \cdot x + 7 \cdot x)}{810000}\right)\right) - \frac{1300}{21} \cdot \sin\left(\operatorname{atan}\left(-\frac{27000 \cdot (-73 + 7 \cdot x + 7 \cdot x)}{810000}\right)\right)$

$Vbc(x) := Vbco(x) \cdot \cos(\alpha(x)) - H \cdot \sin(\alpha(x))$

$Vbc(x) = \left(-\frac{10 \cdot (-93 + 7 \cdot x + 7 \cdot x + 28)}{7}\right) \cdot \cos\left(\operatorname{atan}\left(-\frac{27000 \cdot (-73 + 7 \cdot x + 7 \cdot x)}{810000}\right)\right) - \frac{1300}{21} \cdot \sin\left(\operatorname{atan}\left(-\frac{27000 \cdot (-73 + 7 \cdot x + 7 \cdot x)}{810000}\right)\right)$

* para x = 4 teremos cortes diferentes à esquerda do ponto "b" e à direita do ponto "b":

$Vab(4) = 15,467$ * na esquerda do ponto "b"

$Vbc(4) = -19,334$ * na direita do ponto "b"

Equações de Esforços Axiais do Arco:

$$Nab(x) := (-Vabo(x)) \cdot \sin(\alpha(x)) - H \cdot \cos(\alpha(x))$$

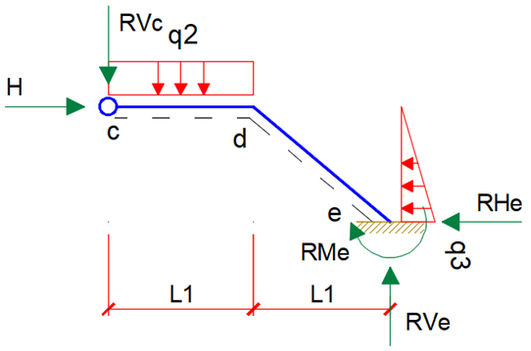
$$Nab(x) = \left(-\left(-\frac{70 \cdot (-93 + 7 \cdot x + 7 \cdot x)}{49}\right)\right) \cdot \sin\left(\operatorname{atan}\left(-\frac{27000 \cdot (-73 + 7 \cdot x + 7 \cdot x)}{810000}\right)\right) - \frac{1300}{21} \cdot \cos\left(\operatorname{atan}\left(-\frac{27000 \cdot (-73 + 7 \cdot x + 7 \cdot x)}{810000}\right)\right)$$

$$Nbc(x) := (-Vbco(x)) \cdot \sin(\alpha(x)) - H \cdot \cos(\alpha(x))$$

$$Nbc(x) = \left(-\left(-\frac{10 \cdot (-93 + 7 \cdot x + 7 \cdot x + 28)}{7}\right)\right) \cdot \sin\left(\operatorname{atan}\left(-\frac{27000 \cdot (-73 + 7 \cdot x + 7 \cdot x)}{810000}\right)\right) - \frac{1300}{21} \cdot \cos\left(\operatorname{atan}\left(-\frac{27000 \cdot (-73 + 7 \cdot x + 7 \cdot x)}{810000}\right)\right)$$

$Nbc(7) = -77,737$

Esforços Cortantes e Axiais na Barra "de":



Força Vertical e Horizontal no ponto "d":

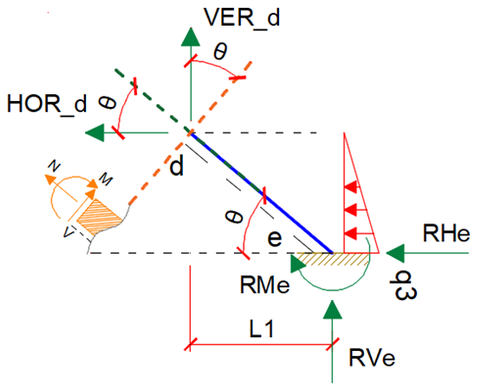
$$VER_d := -RVc - q2 \cdot L1 = -147,14$$

$$HOR_d := -H = -61,905$$

$$Lde := \sqrt{L1^2 + H2^2} = 5$$

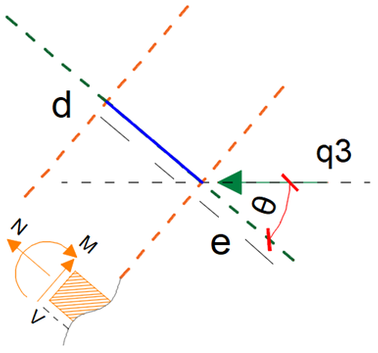
$$cos\theta := \frac{L1}{Lde} = 0,8$$

$$sen\theta := \frac{H2}{Lde} = 0,6$$



$$Vd := VER_d \cdot cos\theta - HOR_d \cdot sen\theta = -80,571$$

$$Nd := VER_d \cdot sen\theta + HOR_d \cdot cos\theta = -137,81$$



$$q3_perp_e := q3 \cdot \frac{H2}{Lde} \cdot sen\theta = 10,8 \qquad q3_para_e := q3 \cdot \frac{H2}{Lde} \cdot cos\theta = 14,4$$

$$qde_perp(x) := \frac{q3_perp_e}{Lde} \cdot x \qquad qde_para(x) := \frac{q3_para_e}{Lde} \cdot x$$

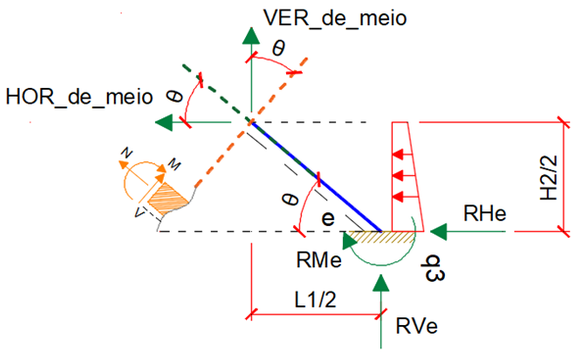
$$Vde(x) := Vd - \int_0^x qde_perp \, d\, x$$

$$Vde(x) := Vd - \frac{q3_perp_e}{Lde} \cdot \frac{x^2}{2} = -\frac{3 \cdot \left(4700 + 63 \cdot x^2\right)}{175}$$

$$Nde(x) := Nd + \int_0^x qde_para \, d\, x$$

$$Nde(x) := Nd + \frac{q3_para_e}{Lde} \cdot \frac{x^2}{2} = \frac{2 \cdot \left(-36175 + 378 \cdot x^2\right)}{525}$$

Forma Alternativa de Cálculo dos Esforços Cortantes e Axiais no meio da Barra "de":



Força Vertical e Horizontal no meio da barra "de":

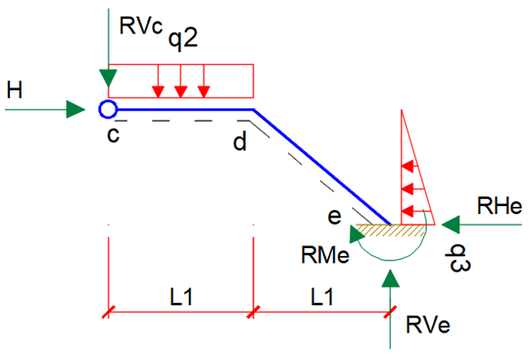
$$VER_de_meio := -RVc - q2 \cdot L1 = -147,1429$$

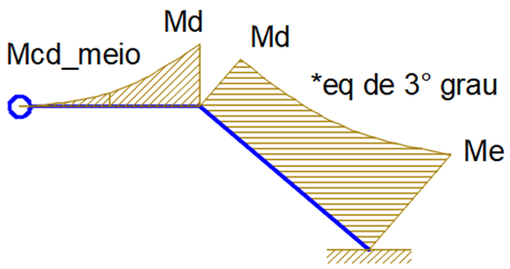
$$HOR_de_meio := -H + \frac{H2}{2} \cdot \frac{q3}{2} \cdot \frac{1}{2} = -50,6548$$

$$V_de_meio := VER_de_meio \cdot cos\theta - HOR_de_meio \cdot sen\theta = -87,321 \qquad Vde\left(\frac{Lde}{2}\right) = -87,321$$

$$N_de_meio := VER_de_meio \cdot sen\theta + HOR_de_meio \cdot cos\theta = -128,81 \qquad Nde\left(\frac{Lde}{2}\right) = -128,81$$

Diagrama de Momentos do Carregamento Real em "cde":





$$RVc = 87,143$$

$$H = 61,905$$

$$q2 = 15$$

$$Mc := 0$$

$$Md := -RVc \cdot L1 - \frac{q2 \cdot L1^{2}}{2} = -468,57 \qquad Mcd_meio := \frac{Mc + Md}{2} + \frac{q2 \cdot L1^{2}}{8} = -204,29$$

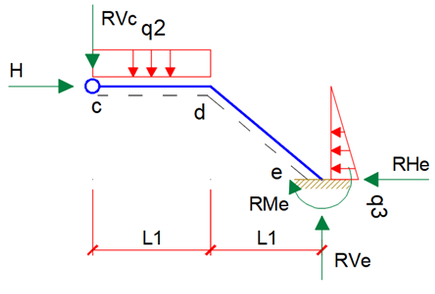
* Como o trecho "de" possui uma carga distribuída variando linearmente, o diagrama de momentos é parabólico de 3° grau, o que implica na impossibilidade de uso da tabela de integrais geométricas para este trecho.

$$qde_perp(x) := \frac{q3_perp_e}{Lde} \cdot x$$

$$Mde(x) := Md + Vd \cdot x - \int_0^x \int_0^x qde_perp(x)\,dx\,dx$$

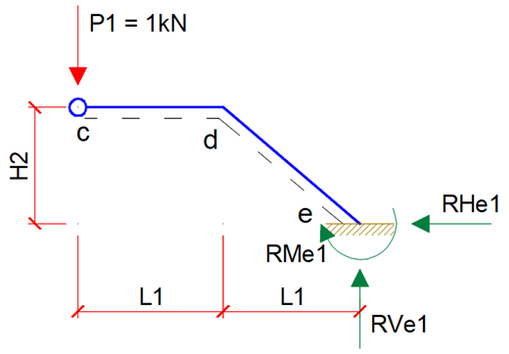
$$Mde(x) := Md + Vd \cdot x - \frac{q3_perp_e}{Lde} \cdot \frac{x^{3}}{6} = -\frac{100 \cdot (820 + 141 \cdot x) + 63 \cdot x^{3}}{175} \qquad Me := Mde(Lde) = -916,43$$

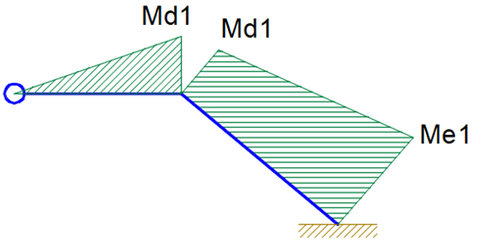
Verificando o cálculo da reação de apoio de momento fletor no ponto "e" para conferir com o resultado da equação "Mde(x)":



$$Me_teste := H \cdot H2 - RVc \cdot (L1 + L1) - q2 \cdot L1 \cdot \left(\frac{L1}{2} + L1\right) - q3 \cdot H2 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot H2 = -916,43$$

Diagrama de Momentos do Carregamento Virtual em "cde":





$P1 := 1$ $\quad Md1 := -P1 \cdot L1 = -4$ $\quad Me1 := -P1 \cdot (L1 + L1) = -8$

Equacionando o trecho "de" para proceder a integração analítica no trecho:

$Mde1(x) := Md1 - P1 \cdot cos\theta \cdot x = -4 - 1 \cdot \frac{4}{5} \cdot x$

$Mde1(0) = -4$ $\quad Mde1(Lde) = -8$

Cálculo do Deslocamento Vertical em "c":

Diagrama M Real

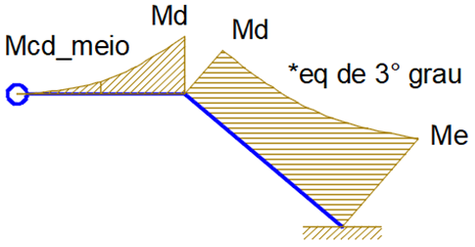
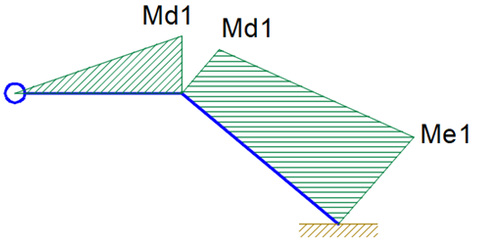
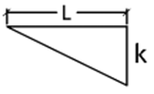


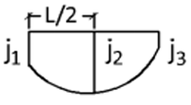
Diagrama M Virtual



$EI := \frac{E \cdot b \cdot h^3}{12} = 4,2667 \cdot 10^5$

Barra "cd":





1/6 Lk(2j2+j3)

$k := Md1 = -4$ $\quad L := L1 = 4$

$j2 := Mcd_meio = -204,29$ $\quad j3 := Md = -468,57$

$mcd := \frac{1}{6} \cdot L \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{EI} = 0,0054821$

Barra "de" : - devido ao formato parabólico de 3° grau do diagrama de momentos real é impossível a utilização da tabela de integração disponibilizada, logo, o procedimento será feito via integração analítica.

$mde := \frac{1}{EI} \cdot \int_0^{Lde} Mde1(x) \cdot Mde(x) \, dx = 0,049632$

$\delta_ver_c := -1000 \cdot (mcd + mde) = -55,114$ mm

$Q01 := Mab(3) = -13,333$

$Q02 := Vab(4) = 15,467$

$Q03 := Nbc(7) = -77,737$

$Q04 := Vde\left(\frac{Lde}{2}\right) = -87,321$

$Q05 := Nde\left(\frac{Lde}{2}\right) = -128,81$

$Q06 := \delta_ver_c = -55,114$
