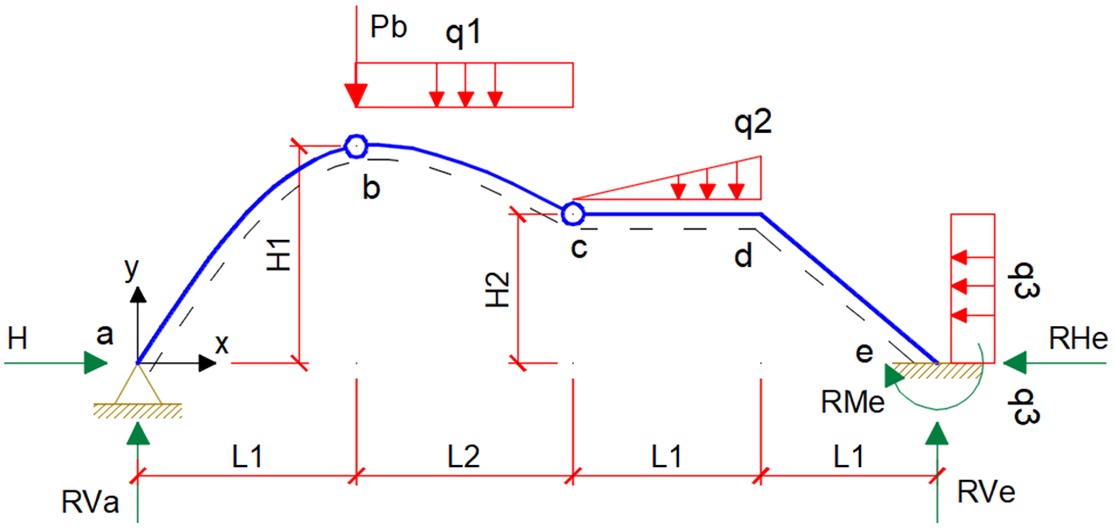
B2T4NA0622A

- Analisando o modelo estrutural abaixo, responda os questionamentos.



- Dados:

$L1 := 5,0$ m $L2 := 4,0$ m $H1 := 6,0$ m $H2 := 4,0$ m

$q1 := 20$ kN/m $q2 := 40$ kN/m $q3 := 20$ kN/m $Pb := 40$ kN

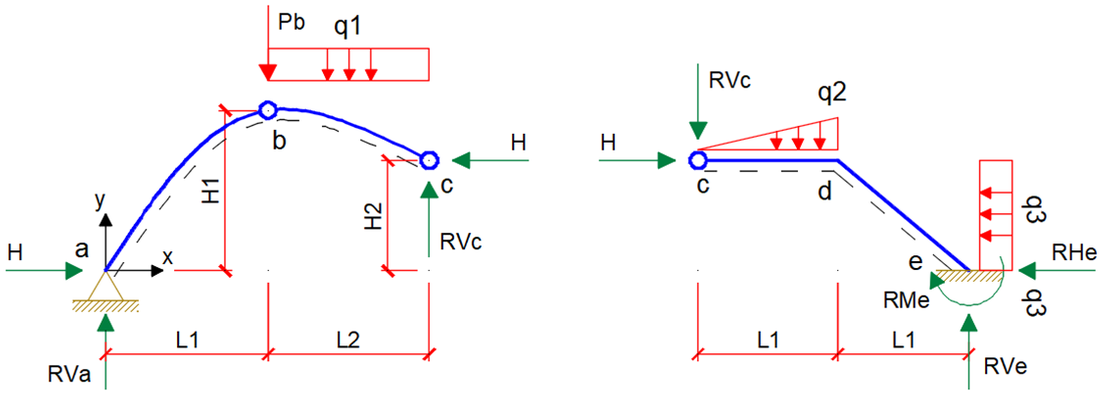
- Todas as barras são de seção transversal retangular de base "b", altura "h" e Módulo de Elasticidade "E":

$b := 0,20$ m $h := 0,80$ m $E := 40 \cdot 10^{6}$ kN/m²

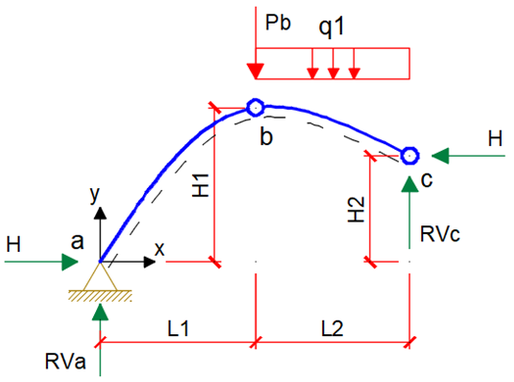
- O arco "abc" é parabólico de 2° grau.

Q01 - Qual o valor do momento fletor no arco "abc" na coordenada x = 3,0 m [em kNm e sinal conforme a convenção] ?
Q02 - Qual o valor do esforço cortante no arco "abc" na coordenada x = 4,0 m [em kN e sinal conforme a convenção] ?
Q03 - Qual o valor do esforço axial no arco "abc" na coordenada x = 8,0 m [em kN e sinal conforme a convenção] ?
Q04 - Qual o valor do esforço cortante no meio da barra "de" [em kN e sinal conforme a convenção] ?
Q05 - Qual o valor do esforço axial no meio da barra "de" [em kN e sinal conforme a convenção] ?
Q06 - Considerando somente efeitos de momentos fletores, qual o valor do deslocamento vertical do ponto "c" [em "mm" e com positivo para cima] ?

Decomposição da Estrutura:



Equacionamento da Geometria do Arco "abc" :



$xa := 0$ $\quad ya := 0$

$xb := L1 = 5$ $\quad yb := H1 = 6$

$xc := L1 + L2 = 9$ $\quad yc := H2 = 4$

$$matrizA := \begin{bmatrix} 1 & xa & xa^2 \\ 1 & xb & xb^2 \\ 1 & xc & xc^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 5 & 25 \\ 1 & 9 & 81 \end{bmatrix} \qquad vetorB := \begin{bmatrix} ya \\ yb \\ yc \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 4 \end{bmatrix}$$

$$vetorX := matrizA^{-1} \cdot vetorB = \begin{bmatrix} 0 \\ 2,1444 \\ -0,1889 \end{bmatrix}$$

$A := vetorX_1 = 0$

$B := vetorX_2 = 2,1444$

$C := vetorX_3 = -0,1889$

$$y(x) := A + B \cdot x + C \cdot x^2 = 0 + \frac{193}{90} \cdot x + \left(-\frac{17}{90}\right) \cdot x^2$$

Reações de Apoio do Arco "abc" :

ΣMc = 0 (+) horário - olhando para "a"

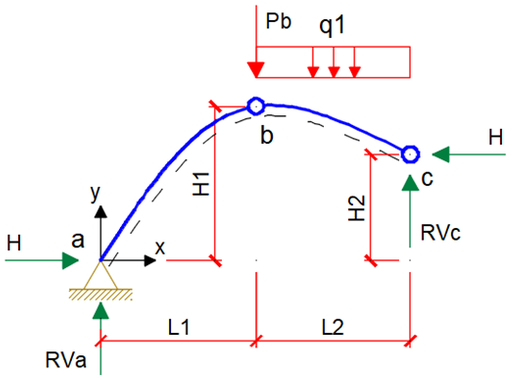
```
RVa.(L1+L2) - H.H2 - Pb.L2 - q1.L2^2 / 2 = 0
```

ΣMb = 0 (+) horário - olhando para "a"

```
RVa.L1 - H.H1 =  0
```

Sistema de equações:

```
RVa.(L1+L2) - H.H2 = Pb.L2 + q1.L2^2 / 2
RVa.L1 - H.H1 = 0
```



$$matrizA := \begin{bmatrix} L1 + L2 & -H2 \\ L1 & -H1 \end{bmatrix} = \begin{bmatrix} 9 & -4 \\ 5 & -6 \end{bmatrix} \qquad vetorB := \begin{bmatrix} \frac{q1 \cdot L2^2}{2} + Pb \cdot L2 \\ 0 \end{bmatrix} = \begin{bmatrix} 320 \\ 0 \end{bmatrix}$$

$$vetorX := matrizA^{-1} \cdot vetorB = \begin{bmatrix} 56,471 \\ 47,059 \end{bmatrix}$$

$RVa := vetorX_1 = 56,471$

$H := vetorX_2 = 47,059$

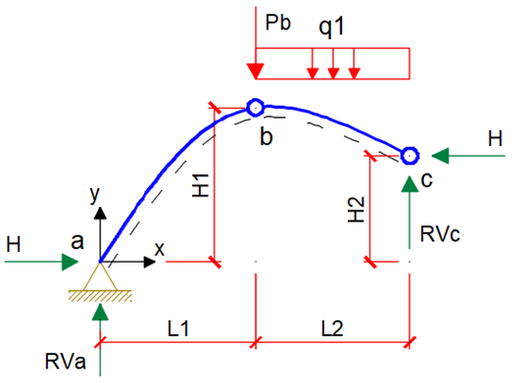
ΣFh = 0 (+) para direita: H igual em módulo porém com sentido contrário.

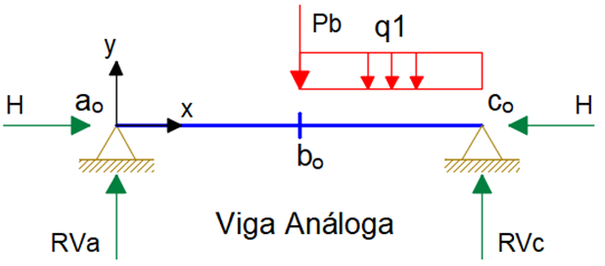
ΣFv = 0 (+) para cima

```
RVa + RVc - q1.L2 - Pb = 0
```

$RVc := -RVa + q1 \cdot L2 + Pb = 63,529$

Equacionamento dos Esforços Internos no Arco "abc":





Equações de Momentos Fletores da Viga Análoga:

Trecho ab: $Mabo(x) := RVa \cdot x = \frac{960}{17} \cdot x$

Trecho bc: $Mbco(x) := RVa \cdot x - \frac{q1 \cdot (x - L1)^2}{2} - Pb \cdot (x - L1) = \frac{960}{17} \cdot x - \frac{20 \cdot (x - 5)^2}{2} - 40 \cdot (x - 5)$

Equação de Esforços Cortantes da Viga Análoga:

Trecho ab: $Vabo(x) := \frac{d}{dx} Mabo(x) = \frac{960}{17}$

Trecho bc: $Vbco(x) := \frac{d}{dx} Mbco(x) = -\frac{20 \cdot (-14 + 17 \cdot (-5 + x))}{17}$

Equação de Momentos Fletores do Arco:

$Mab(x) := Mabo(x) - H \cdot y(x)$

$Mab(x) = \frac{960}{17} \cdot x - \frac{800}{17} \cdot \left(0 + \frac{193}{90} \cdot x + \left(-\frac{17}{90}\right) \cdot x^2\right)$ $\qquad Mab(3) = -53,333$

$Mbc(x) := Mbco(x) - H \cdot y(x)$

$Mbc(x) = \frac{960}{17} \cdot x - \frac{20 \cdot (x - 5)^2}{2} - 40 \cdot (x - 5) - \frac{800}{17} \cdot \left(0 + \frac{193}{90} \cdot x + \left(-\frac{17}{90}\right) \cdot x^2\right)$

$$V_s(x) = V_{so}(x).cos(\alpha(x)) - H.sen(\alpha(x))$$
$$N_s(x) = -V_{so}(x).sen(\alpha(x)) - H.cos(\alpha(x))$$

$$\alpha(x) = arctg\left(\frac{dy(x)}{dx}\right)$$

$\alpha(x) := \operatorname{atan}\left(\frac{d}{dx} y(x)\right) = -\operatorname{atan}\left(\frac{-193 + 34 \cdot x}{90}\right)$

Equações de Esforços Cortantes do Arco:

$Vab(x) := Vabo(x) \cdot \cos(\alpha(x)) - H \cdot \sin(\alpha(x))$

$Vab(x) = \frac{960}{17} \cdot \cos\left(\operatorname{atan}\left(-\frac{729000 \cdot (-193 + 17 \cdot x + 17 \cdot x)}{65610000}\right)\right) - \frac{800}{17} \cdot \sin\left(\operatorname{atan}\left(-\frac{729000 \cdot (-193 + 17 \cdot x + 17 \cdot x)}{65610000}\right)\right)$

$Vbc(x) := Vbco(x) \cdot \cos(\alpha(x)) - H \cdot \sin(\alpha(x))$

$Vbc(x) = \left(-\frac{20 \cdot (-48 + 17 \cdot (-5 + x) + 34)}{17}\right) \cdot \cos\left(\operatorname{atan}\left(-\frac{729000 \cdot (-193 + 17 \cdot x + 17 \cdot x)}{65610000}\right)\right) - \frac{800}{17} \cdot \sin\left(\operatorname{atan}\left(-\frac{729000 \cdot (-193 + 17 \cdot x + 17 \cdot x)}{65610000}\right)\right)$

$Vab(4) = 22,529$

Equações de Esforços Axiais do Arco:

$$Nab(x) := (-Vabo(x)) \cdot \sin(\alpha(x)) - H \cdot \cos(\alpha(x))$$

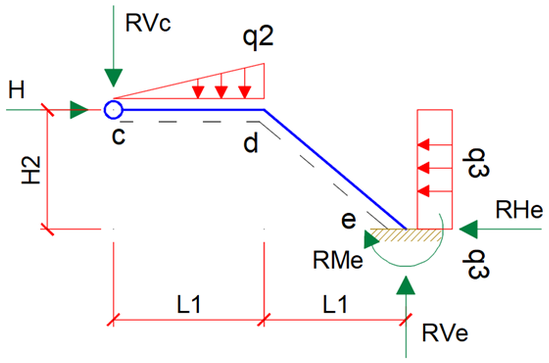
$$Nab(x) = \left(-\frac{960}{17}\right) \cdot \sin\left(\operatorname{atan}\left(-\frac{729000 \cdot (-193 + 17 \cdot x + 17 \cdot x)}{65610000}\right)\right) - \frac{800}{17} \cdot \cos\left(\operatorname{atan}\left(-\frac{729000 \cdot (-193 + 17 \cdot x + 17 \cdot x)}{65610000}\right)\right)$$

$$Nbc(x) := (-Vbco(x)) \cdot \sin(\alpha(x)) - H \cdot \cos(\alpha(x))$$

$$Nbc(x) = \left(-\left(-\frac{20 \cdot (-48 + 17 \cdot (-5 + x) + 34)}{17}\right)\right) \cdot \sin\left(\operatorname{atan}\left(-\frac{729000 \cdot (-193 + 17 \cdot x + 17 \cdot x)}{65610000}\right)\right) - \frac{800}{17} \cdot \cos\left(\operatorname{atan}\left(-\frac{729000 \cdot (-193 + 17 \cdot x + 17 \cdot x)}{65610000}\right)\right)$$

$$Nbc(8) = -64,082$$

Esforços Cortantes e Axiais na Barra "de":



Força Vertical e Horizontal no ponto "d":

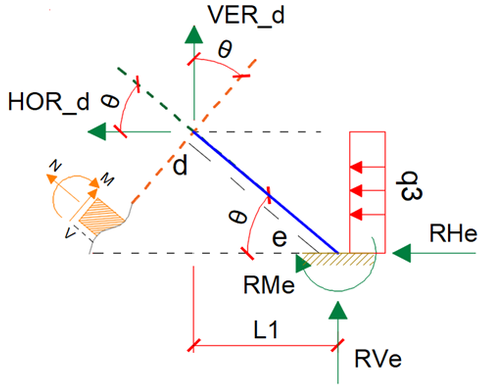
$$VER_d := -RVc - q2 \cdot L1 \cdot \frac{1}{2} = -163,53$$

$$HOR_d := -H = -47,059$$

$$Lde := \sqrt{L1^{2} + H2^{2}} = 6,4031$$

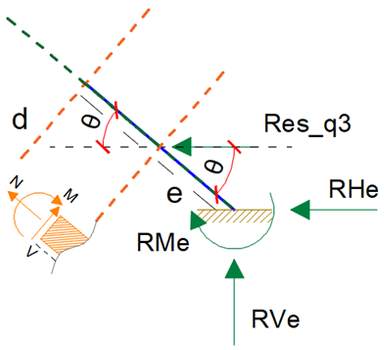
$$cos\theta := \frac{L1}{Lde} = 0,78087$$

$$sen\theta := \frac{H2}{Lde} = 0,6247$$



$$Vd := VER_d \cdot cos\theta - HOR_d \cdot sen\theta = -98,298$$

$$Nd := VER_d \cdot sen\theta + HOR_d \cdot cos\theta = -138,90$$



$$Res_q3 := q3 \cdot H2 = 80$$

$$qde_perp := \frac{Res_q3 \cdot sen\theta}{Lde} = 7,8049$$

$$qde_para := \frac{Res_q3 \cdot cos\theta}{Lde} = 9,7561$$

$$Vde(x) := Vd - \int_{0}^{x} qde_perp \, dx$$

$$Vde(x) := Vd - qde_perp \cdot x$$

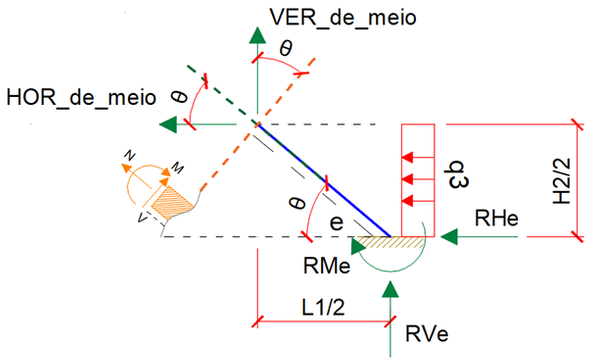
$$Vde\left(\frac{Lde}{2}\right) = -123,29$$

$$Nde(x) := Nd + \int_{0}^{x} qde_para \, dx$$

$$Nde(x) := Nd + qde_para \cdot x$$

$$Nde\left(\frac{Lde}{2}\right) = -107,668$$

Forma Alternativa de Cálculo dos Esforços Cortante e Axial no meio da Barra "de":



Força Vertical e Horizontal no meio da barra "de":

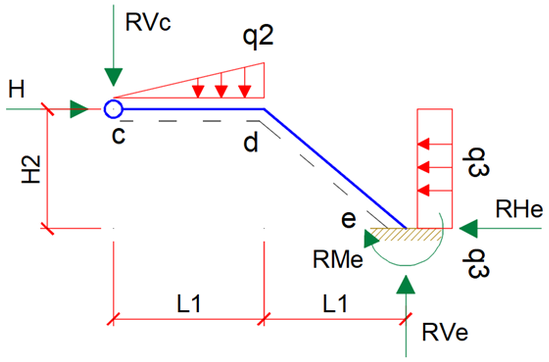
$$VER_de_meio := -RVc - q2 \cdot L1 \cdot \frac{1}{2} = -163,53$$

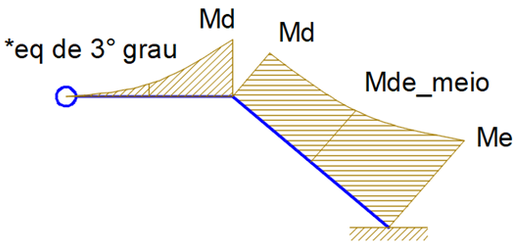
$$HOR_de_meio := -H + q3 \cdot \frac{H2}{2} = -7,0588$$

$$V_de_meio := VER_de_meio \cdot cos\theta - HOR_de_meio \cdot sen\theta = -123,29 \qquad Vde\left(\frac{Lde}{2}\right) = -123,29$$

$$N_de_meio := VER_de_meio \cdot sen\theta + HOR_de_meio \cdot cos\theta = -107,67 \qquad Nde\left(\frac{Lde}{2}\right) = -107,67$$

Diagrama de Momentos do Carregamento Real em "cde":





$$RVc = 63,529$$

$$H = 47,059$$

$$q2 = 40$$

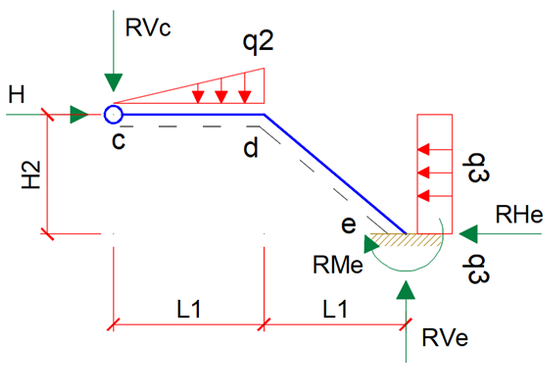
$$Mc := 0$$

$$Md := -RVc \cdot L1 - q2 \cdot L1 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot L1 = -484,31$$

$$Me := -RVc \cdot (L1 + L1) - q2 \cdot L1 \cdot \frac{1}{2} \cdot \left(\frac{1}{3} \cdot L1 + L1\right) + H \cdot H2 - \frac{q3 \cdot H2^{2}}{2} = -1273,7$$

$$Mde_meio := \frac{Md + Me}{2} + \frac{q3 \cdot H2^{2}}{8} = -839,02$$

* Como o trecho "cd" possui uma carga distribuída variando linearmente, o diagrama de momentos é parabólico de 3° grau, o que implica na impossibilidade de uso da tabela de integrais geométricas para este trecho.



$$qcd(x) := \frac{q2}{L1} \cdot x = 8 \cdot x$$

$$Mcd(x) := -RVc \cdot x - \int_{0}^{x}\int_{0}^{x} qcd(x)\,dx\,dx$$

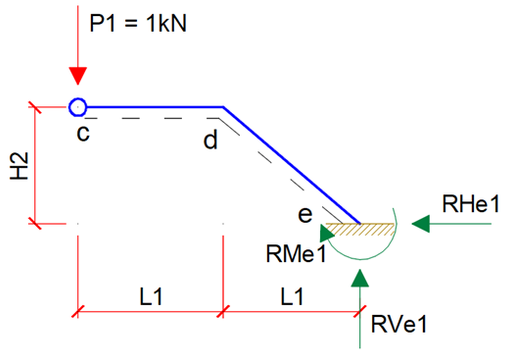
$$Mcd(x) := -RVc \cdot x - \frac{q2}{L1} \cdot \frac{x^{3}}{6} = -\frac{4 \cdot x \cdot \left(810 + 17 \cdot x^{2}\right)}{51}$$

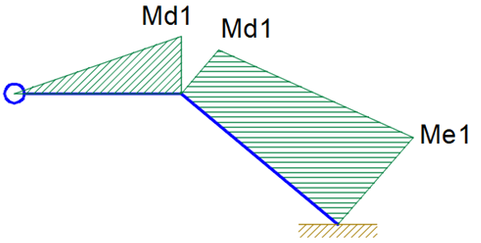
Verificando o cálculo da reação de apoio de momento fletor no ponto "e" para conferir com os resultados:

$$Md_teste := Mcd(L1) = -484,31 \qquad Vcd(x) := \frac{d}{dx} Mcd(x)$$

$$Me_teste := Md_teste + H \cdot H2 - \frac{q3 \cdot H2^{2}}{2} + Vcd(L1) \cdot L1 = -1273,7$$

Diagrama de Momentos do Carregamento Virtual em "cde":





$P1 := 1$ $\quad Md1 := -P1 \cdot L1 = -5$ $\quad Me1 := -P1 \cdot (L1 + L1) = -10$

Equacionando o trecho "cd" para proceder a integração analítica no trecho:

$Mcd1(x) := -P1 \cdot x = -x$

Cálculo do Deslocamento Vertical em "c":

Diagrama M Real

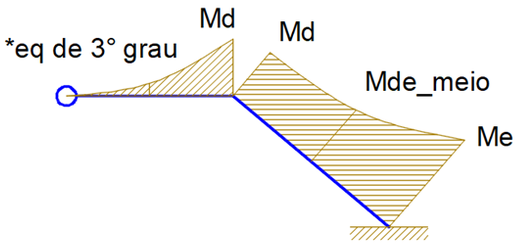
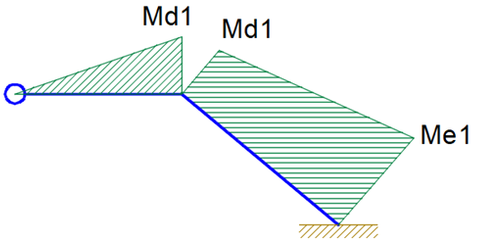


Diagrama M Virtual

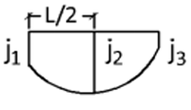


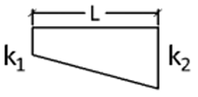
$EI := \frac{E \cdot b \cdot h^3}{12} = 3,4133 \cdot 10^5$

Barra "cd" : - devido ao formato parabólico de 3° grau do diagrama de momentos real é impossível a utilização da tabela de integração disponibilizada, logo, o procedimento será feito via integração analítica.

$mcd := \frac{1}{EI} \cdot \int_0^{L1} Mcd1(x) \cdot Mcd(x) \, dx = 0,010196$

Barra "de":





1/6 L[j_1k_1+ $2j_2(k_1+k_2)+j_3k_2$]

$k1 := Md1 = -5$ $\quad j1 := Md = -484,3137$

$k2 := Me1 = -10$ $\quad j2 := Mde_meio = -839,0196$

$L := Lde = 6,4031$ $\quad j3 := Me = -1273,7255$

$mde := \frac{1}{6} \cdot L \cdot (j1 \cdot k1 + 2 \cdot j2 \cdot (k1 + k2) + j3 \cdot k2) \cdot \frac{1}{EI} = 0,1261$ m

$\delta_ver_c := -1000 \cdot (mcd + mde) = -136,29$ mm

$Q01 := Mab(3) = -53,333$

$Q02 := Vab(4) = 22,529$

$Q03 := Nbc(8) = -64,082$

$Q04 := Vde\left(\frac{Lde}{2}\right) = -123,29$

$Q05 := Nde\left(\frac{Lde}{2}\right) = -107,67$

$Q06 := \delta_ver_c = -136,29$
