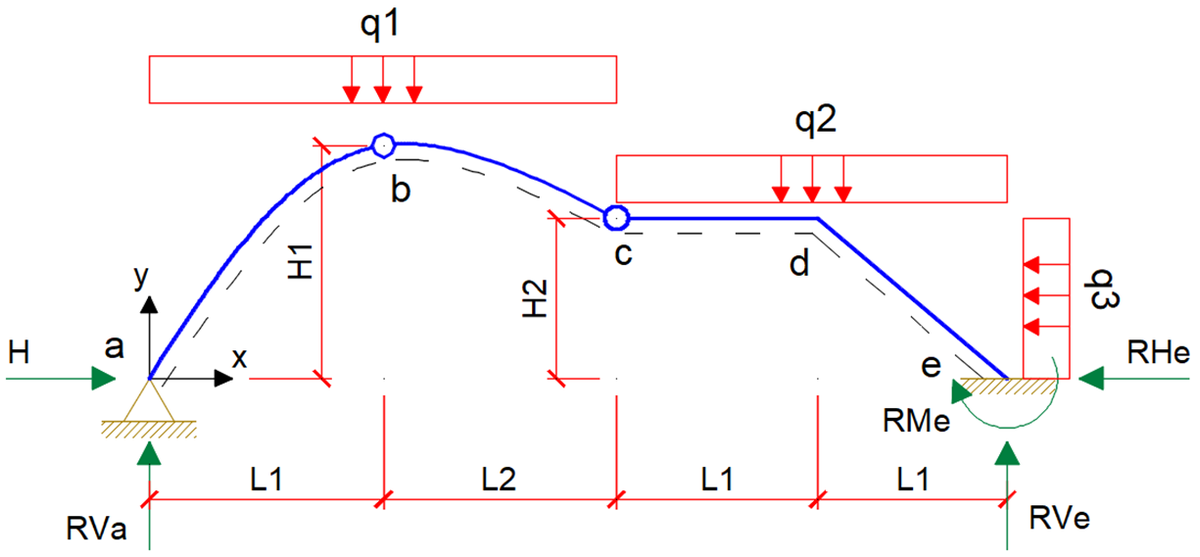
B2T4NB0617A

- Analisando o modelo estrutural abaixo, responda os questionamentos.



- Dados:

$L1 := 4,0\ \text{m}$ $L2 := 5,0\ \text{m}$ $H1 := 6,0\ \text{m}$ $H2 := 3,0\ \text{m}$ $q1 := 20\ \text{kN/m}$ $q2 := 30\ \text{kN/m}$ $q3 := 10\ \text{kN/m}$

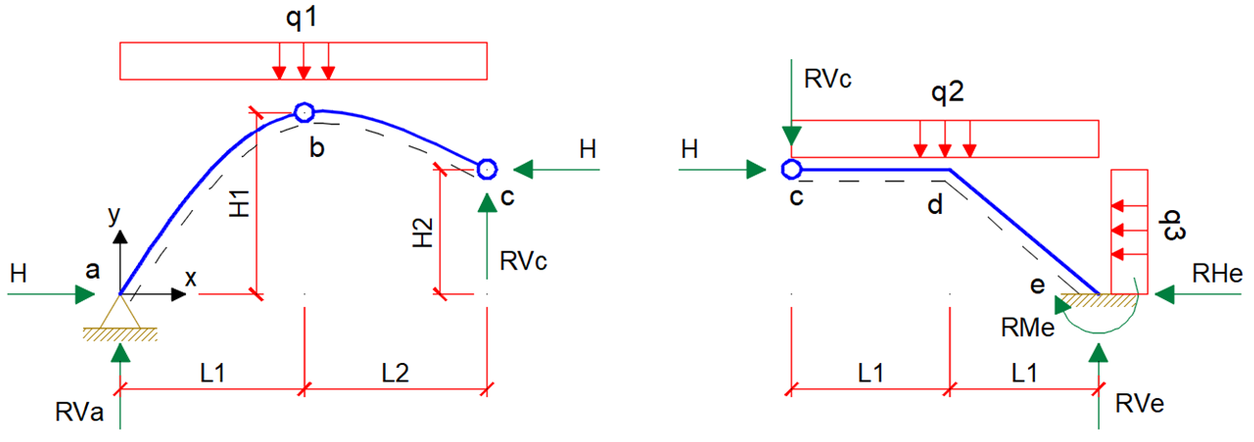
- Todas as barras são de seção transversal retangular de base "b", altura "h" e Módulo de Elasticidade "E":

$b := 0,15\ \text{m}$ $h := 0,75\ \text{m}$ $E := 40 \cdot 10^{6}\ \text{kN/m}^2$

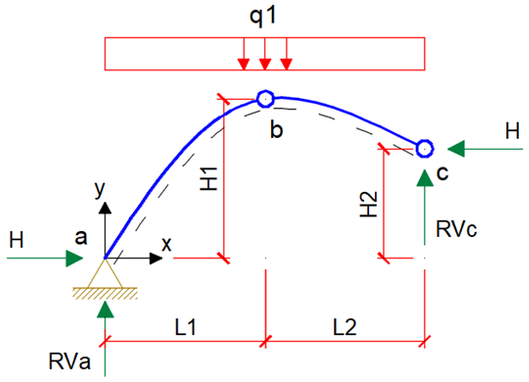
- O arco "abc" é parabólico de 2° grau.

Q01 - Qual o valor do momento fletor no arco "abc" na coordenada x = 3,0 m [em kNm e sinal conforme a convenção] ?
Q02 - Qual o valor do esforço cortante no arco "abc" na coordenada x = 4,0 m [em kN e sinal conforme a convenção] ?
Q03 - Qual o valor do esforço axial no arco "abc" na coordenada x = 7,0 m [em kN e sinal conforme a convenção] ?
Q04 - Qual o valor do esforço cortante no meio da barra "de" [em kN e sinal conforme a convenção] ?
Q05 - Qual o valor do esforço axial no meio da barra "de" [em kN e sinal conforme a convenção] ?
Q06 - Considerando somente efeitos de momentos fletores, qual o valor do deslocamento vertical do ponto "c" [em "mm" e com positivo para cima] ?

Decomposição da Estrutura:



Equacionamento da Geometria do Arco "abc" :



$xa := 0$ $\quad ya := 0$

$xb := L1 = 4$ $\quad yb := H1 = 6$

$xc := L1 + L2 = 9$ $\quad yc := H2 = 3$

$$matrizA := \begin{bmatrix} 1 & xa & xa^2 \\ 1 & xb & xb^2 \\ 1 & xc & xc^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 4 & 16 \\ 1 & 9 & 81 \end{bmatrix} \qquad vetorB := \begin{bmatrix} ya \\ yb \\ yc \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 3 \end{bmatrix}$$

$$vetorX := matrizA^{-1} \cdot vetorB = \begin{bmatrix} 0 \\ 2,4333 \\ -0,2333 \end{bmatrix}$$

$A := vetorX_1 = 0$

$B := vetorX_2 = 2,4333$

$C := vetorX_3 = -0,2333$

$$y(x) := A + B \cdot x + C \cdot x^2 = 0 + \frac{73}{30} \cdot x + \left(-\frac{7}{30}\right) \cdot x^2$$

Reações de Apoio do Arco "abc" :

ΣMc = 0 (+) horário - olhando para "a"

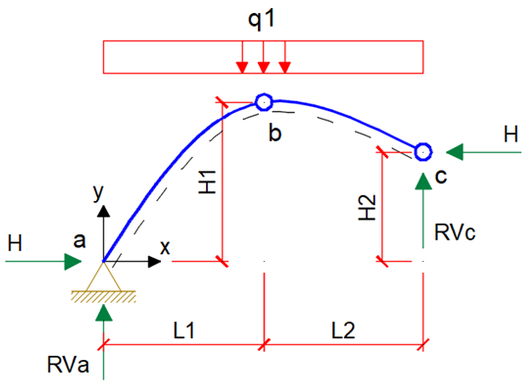
```
RVa.(L1+L2) - H.H2 - q1.(L1+L2)^2 / 2 = 0
```

ΣMb = 0 (+) horário - olhando para "a"

```
RVa.L1 - H.H1 - q1.L1^2 / 2 =  0
```

Sistema de equações:

```
RVa.(L1+L2) - H.H2 = q1.(L1+L2)^2 / 2
RVa.L1 - H.H1 = q1.L1^2 / 2
```



$$matrizA := \begin{bmatrix} L1 + L2 & -H2 \\ L1 & -H1 \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ 4 & -6 \end{bmatrix} \qquad vetorB := \begin{bmatrix} \frac{q1 \cdot (L1 + L2)^2}{2} \\ \frac{q1 \cdot L1^2}{2} \end{bmatrix} = \begin{bmatrix} 810 \\ 160 \end{bmatrix}$$

$$vetorX := matrizA^{-1} \cdot vetorB = \begin{bmatrix} 104,29 \\ 42,857 \end{bmatrix}$$

$RVa := vetorX_1 = 104,29$

$H := vetorX_2 = 42,857$

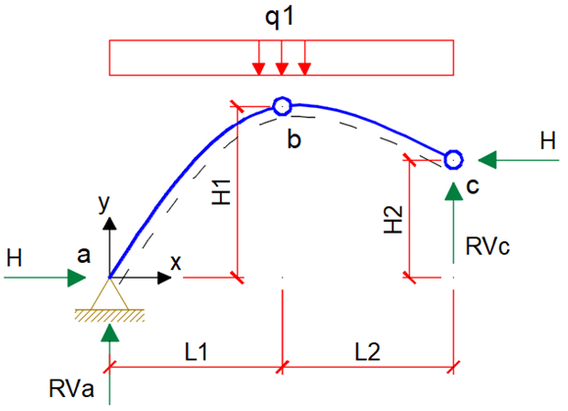
ΣFh = 0 (+) para direita: H igual em módulo porém com sentido contrário.

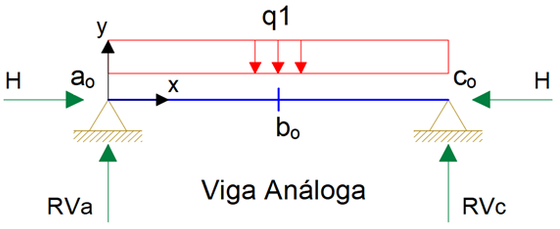
ΣFv = 0 (+) para cima

```
RVa + RVc - q1.(L1+L2) = 0
```

$RVc := -RVa + q1 \cdot (L1 + L2) = 75,714$

Equacionamento dos Esforços Internos no Arco "abc":





Equação de Momentos Fletores da Viga Análoga:

$$Mabco(x) := RVa \cdot x - \frac{q1 \cdot x^2}{2} = \frac{730}{7} \cdot x - \frac{20 \cdot x^2}{2}$$

Equação de Esforços Cortantes da Viga Análoga:

$$Vabco(x) := \frac{d}{dx} Mabco(x) = -\frac{10 \cdot (-73 + 14 \cdot x)}{7}$$

Equação de Momentos Fletores do Arco:

$$Mabc(x) := Mabco(x) - H \cdot y(x)$$

$$Mabc(x) = 0$$

$$\boldsymbol{V_s(x) = V_{so}(x).cos(\alpha(x)) - H.sen(\alpha(x))}$$
$$\boldsymbol{N_s(x) = -V_{so}(x).sen(\alpha(x)) - H.cos(\alpha(x))}$$

$$\boldsymbol{\alpha(x) = arctg\left(\frac{dy(x)}{dx}\right)}$$

$$\alpha(x) := \operatorname{atan}\left(\frac{d}{dx} y(x)\right) = -\operatorname{atan}\left(\frac{-73 + 14 \cdot x}{30}\right)$$

Equação de Esforços Cortantes do Arco:

$$Vabc(x) := Vabco(x) \cdot \cos(\alpha(x)) - H \cdot \sin(\alpha(x))$$

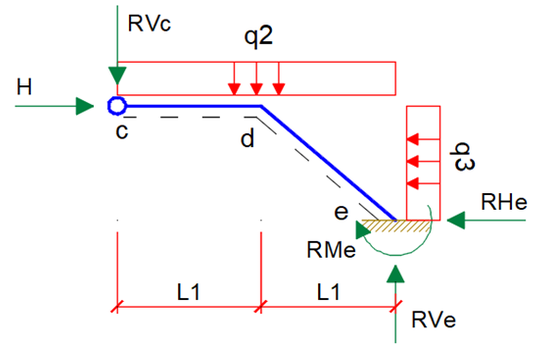
$$Vabc(x) = 0$$

Equação de Esforços Axiais do Arco:

$$Nabc(x) := (-Vabco(x)) \cdot \sin(\alpha(x)) - H \cdot \cos(\alpha(x))$$

$$Nabc(x) = \left(-\left(-\frac{70 \cdot (-73 + 7 \cdot x + 7 \cdot x)}{49}\right)\right) \cdot \sin\left(\operatorname{atan}\left(-\frac{27000 \cdot (-73 + 7 \cdot x + 7 \cdot x)}{810000}\right)\right) - \frac{300}{7} \cdot \cos\left(\operatorname{atan}\left(-\frac{27000 \cdot (-73 + 7 \cdot x + 7 \cdot x)}{810000}\right)\right)$$

Esforços Cortantes e Axiais na Barra "de":



Força Vertical e Horizontal no ponto "d":

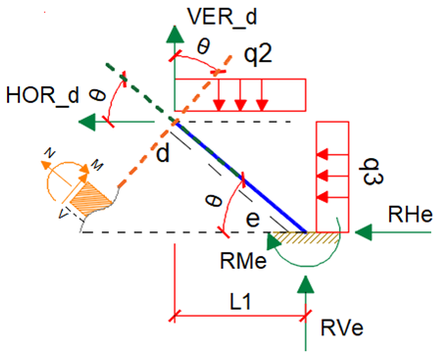
$$VER_d := -RVc - q2 \cdot L1 = -195,71$$

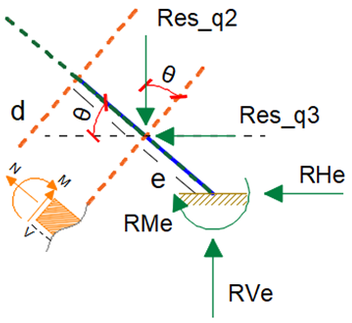
$$HOR_d := -H = -42,857$$

$$Lde := \sqrt{L1^2 + H2^2} = 5$$

$$cos\theta := \frac{L1}{Lde} = 0,8$$

$$sen\theta := \frac{H2}{Lde} = 0,6$$





$Vd := VER_d \cdot cos\theta - HOR_d \cdot sen\theta = -130,86$

$Nd := VER_d \cdot sen\theta + HOR_d \cdot cos\theta = -151,71$

$Res_q2 := q2 \cdot L1 = 120$

$Res_q3 := q3 \cdot H2 = 30$

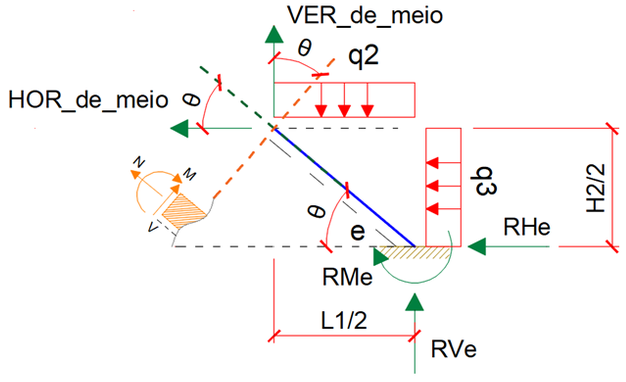
$$q_per := \frac{-Res_q2 \cdot cos\theta - Res_q3 \cdot sen\theta}{Lde} = -22,8$$

$$q_par := \frac{-Res_q2 \cdot sen\theta + Res_q3 \cdot cos\theta}{Lde} = -9,6$$

$$Vde(x) := Vd + q_per \cdot x = -\frac{916}{7} + \left(-\frac{114}{5}\right) \cdot x$$

$$Nde(x) := Nd + q_par \cdot x = -\frac{1062}{7} + \left(-\frac{48}{5}\right) \cdot x$$

Forma Alternativa de Cálculo dos Esforços Cortantes e Axiais no meio da Barra "de":



Força Vertical e Horizontal no meio da barra "de":

$$VER_de_meio := -RVc - q2 \cdot \left(L1 + \frac{L1}{2}\right) = -255,7143$$

$$HOR_de_meio := -H + q3 \cdot \frac{H2}{2} = -27,8571$$

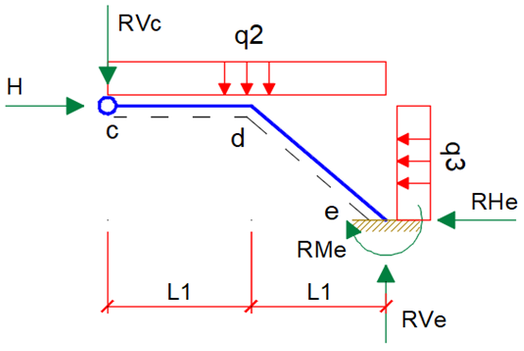
$V_de_meio := VER_de_meio \cdot cos\theta - HOR_de_meio \cdot sen\theta = -187,86$

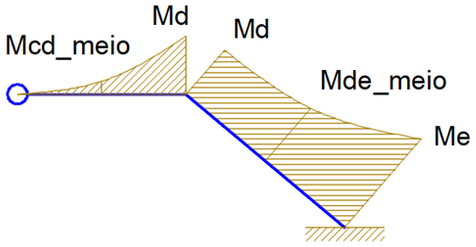
$N_de_meio := VER_de_meio \cdot sen\theta + HOR_de_meio \cdot cos\theta = -175,71$

$$Vde\left(\frac{Lde}{2}\right) = -187,86$$

$$Nde\left(\frac{Lde}{2}\right) = -175,71$$

Diagrama de Momentos do Carregamento Real em "cde":





$RVc = 75,714$

$H = 42,857$

$q2 = 30$

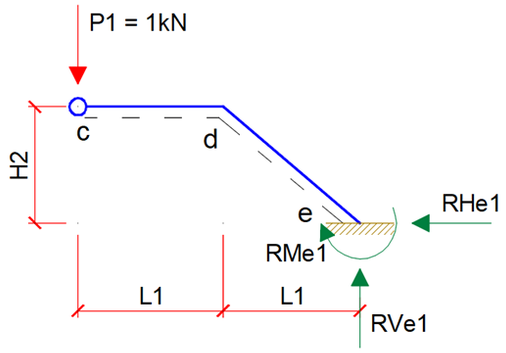
$$Md := -RVc \cdot L1 - \frac{q2 \cdot L1^{2}}{2} = -542,86$$

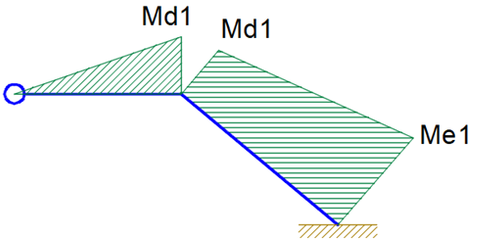
$$Me := -RVc \cdot 2 \cdot L1 - \frac{q2 \cdot (2 \cdot L1)^{2}}{2} - \frac{q3 \cdot H2^{2}}{2} + H \cdot H2 = -1482,1$$

$Mc := 0$

$Mcd_meio := \frac{Mc + Md}{2} + \frac{q2 \cdot L1^{2}}{8} = -211,43$ $\qquad$ $Mde_meio := \frac{Md + Me}{2} + \frac{q2 \cdot L1^{2}}{8} + \frac{q3 \cdot H2^{2}}{8} = -941,25$

Diagrama de Momentos do Carregamento Virutal em "cde":





$P1 := 1$ $\qquad$ $Md1 := -P1 \cdot L1 = -4$ $\qquad$ $Me1 := -P1 \cdot (L1 + L1) = -8$

Cálculo do Deslocamento Vertical em "c":

Diagrama M Real

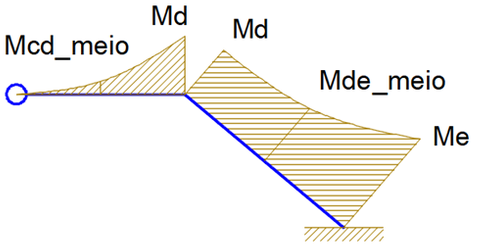
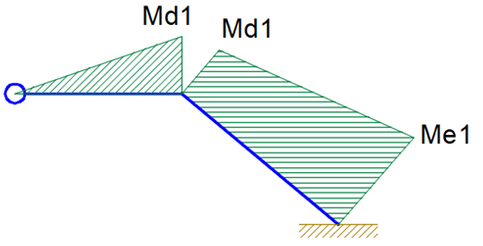
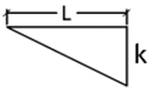


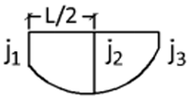
Diagrama M Virtual



$EI := \frac{E \cdot b \cdot h^{3}}{12} = 2,1094 \cdot 10^{5}$

Barra "cd":





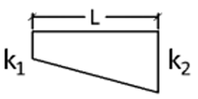
1/6 Lk(2j2+j3)

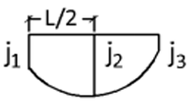
$k := Md1 = -4$ $\qquad$ $L := L1 = 4$

$j2 := Mcd_meio = -211,43$ $\qquad$ $j3 := Md = -542,86$

$mcd := \frac{1}{6} \cdot L \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{EI} = 0,012209$ m

Barra "de":





1/6 L[j_1k_1+ $2j_2(k_1+k_2)+j_3k_2$]

$k1 := Md1 = -4$ $\qquad$ $j1 := Md = -542,86$

$k2 := Me1 = -8$ $\qquad$ $j2 := Mde_meio = -941,25$

$L := Lde = 5$ $\qquad$ $j3 := Me = -1482,1$

$mde := \frac{1}{6} \cdot L \cdot (j1 \cdot k1 + 2 \cdot j2 \cdot (k1 + k2) + j3 \cdot k2) \cdot \frac{1}{EI} = 0,14467$ m

$\delta_ver_c := -1000 \cdot (mcd + mde) = -156,87$ mm

$Q01 := Mabc(3) = 0,0000$

$Q02 := Vabc(4) = 4,64 \cdot 10^{-15}$

$Q03 := Nabc(7) = -55,787$

$Q04 := Vde\left(\frac{Lde}{2}\right) = -187,86$

$Q05 := Nde\left(\frac{Lde}{2}\right) = -175,71$

$Q06 := \delta_ver_c = -156,87$
