B1T4MA0831A

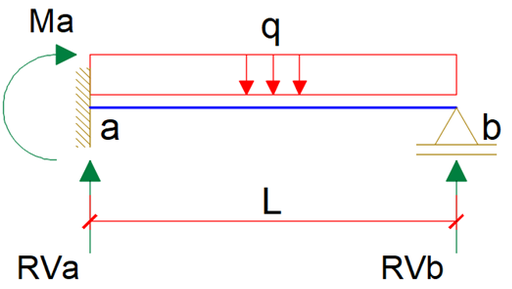
Empregando o método das forças calcule o máximo momento fletor que ocorre na viga respondendo os questionamentos.

* Adotar obrigatoriamente a reação de apoio vertical no ponto "b" como redudante hiperestática.

Q01) Qual o valor do deslocamento vertical que ocorre no ponto "b" do caso 0 de carregamento [em m e em módulo]?
Q02) Qual o valor do deslocamento vertical que ocorre no ponto "b" do caso 1 de carregamento [em m e em módulo]?
Q03) Qual o valor da reação vertical no ponto "b" do caso real [kN positivo para cima]?
Q04) Qual o máximo momento fletor que ocorre na viga [em kNm]?

$q := 20,0 \ \frac{kN}{m}$ $L := 7,0 \ m$

$E := 20 \ GPa$

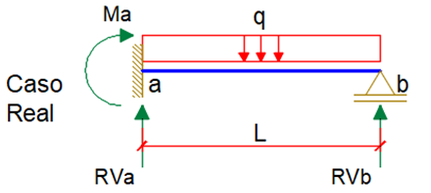


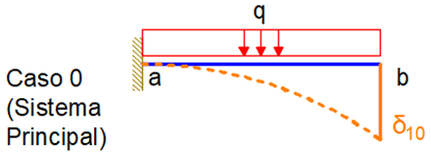
Dimensões da seção transversal retangular:

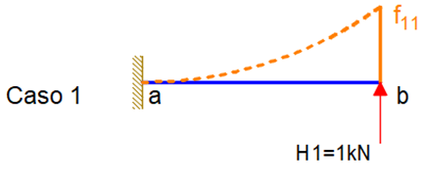
$b := 20 \ cm$ $h := 40 \ cm$

** Considerar nos cálculos somente efeitos de flexão;

Solução via Método das Forças (MF) com Integração da Linha Elástica:







$EI := \frac{E \cdot b \cdot h^3}{12} = 21333 \ kN \ m^2$

Condição e Compatibilidade: $\delta_{10} + RVb \cdot f_{11} = 0$

Caso 0:

$Mba0(x) = -\frac{q \cdot x^2}{2}$

$\theta ba0(x) = \frac{1}{EI} \cdot \left(\int Mba(x) \, dx + \theta b \right)$ ---> $\theta ba0(x) = \frac{1}{EI} \cdot \left(-\frac{q \cdot x^3}{6} + \theta b \right)$

$\theta ba0(L) = 0$ ---> $0 = \frac{1}{EI} \cdot \left(-\frac{q \cdot L^3}{6} + \theta b0 \right)$ ---> $\theta b0 := \frac{q \cdot L^3}{6}$

$\delta ba0(x) = \int \theta ba0(x) \, dx + \delta b0$

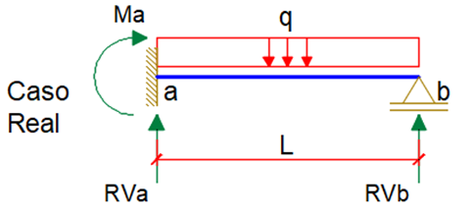
$\delta ba0(x) = \frac{1}{EI} \cdot \left(-\frac{q \cdot x^4}{24} + \theta b0 \cdot x + \delta b0 \right)$

$\delta ba0(L) = 0$ ---> $0 = \frac{1}{EI} \cdot \left(-\frac{q \cdot L^4}{24} + \frac{q \cdot L^3}{6} \cdot L + \delta b0 \right)$

$\delta b0 = -\frac{q \cdot L^4}{6} + \frac{q \cdot L^4}{24}$ ---> $\delta b0 = q \cdot L^4 \cdot \left(\frac{1}{24} - \frac{1}{26} \right)$ ---> $\delta b0 := -\frac{q \cdot L^4}{8}$

$\delta ba0(x) := \frac{1}{EI} \cdot \left(-\frac{q \cdot x^4}{24} + \frac{q \cdot L^3}{6} \cdot x - \frac{q \cdot L^4}{8} \right)$

$\delta_{10} := \delta ba0(0) = -0,28137\ m$ $\quad \frac{\delta b0}{EI} = -0,28137\ m$ $\quad Q01 := |\delta_{10}| = 0,28137\ m$



Caso 1: $H1 := 1\ kN$

$Mba1(x) = H1 \cdot x$

$\theta ba1(x) = \frac{1}{EI} \cdot \left(\int Mba1(x)\,dx + \theta b1 \right)$

$\theta ba1(x) = \frac{1}{EI} \cdot \left(\frac{H1 \cdot x^2}{2} + \theta b1 \right)$

$\theta ba1(L) = 0 \quad ---> \quad \theta b1 = -\frac{H1 \cdot L^2}{2}$

$\delta ba1(x) = \int \theta ba1(x)\,dx + \delta b1$

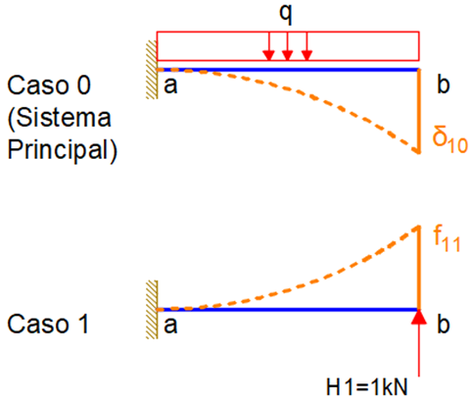
$\delta ba1(x) = \frac{1}{EI} \cdot \left(\frac{H1 \cdot x^3}{6} + \theta b1 \cdot x + \delta b1 \right)$

$\delta ba1(L) = 0 \quad ---> \quad 0 = \frac{1}{EI} \cdot \left(\frac{H1 \cdot L^3}{6} - \frac{H1 \cdot L^2}{2} \cdot L + \delta b1 \right)$

$\delta b1 = -\frac{H1 \cdot L^3}{6} + \frac{H1 \cdot L^3}{2} \quad ---> \quad \delta b1 = H1 \cdot L^3 \cdot \left(-\frac{1}{6} + \frac{1}{2} \right)$

$\delta b1 := \frac{H1 \cdot L^3}{3}$

$\delta ba1(x) := \frac{1}{EI} \cdot \left(\frac{H1 \cdot x^3}{6} - \frac{H1 \cdot L^2}{2} \cdot x + \frac{H1 \cdot L^3}{3} \right)$



$f_{11} := \frac{\delta ba1(0)}{1\ kN} = 0,0053594\ \frac{m}{kN}$ $\quad \frac{H1 \cdot L^3}{3 \cdot EI} = 0,0053594\ m$ $\quad Q02 := |f_{11}|$

Condição e Compatibilidade: $\delta_{10} + RVb \cdot f_{11} = 0$ $\quad --->$ $\quad RVb := \frac{-\delta_{10}}{f_{11}} = 52,5\ kN$ $\quad Q03 := RVb = 52,5\ kN$

Equação de momentos fletores do caso real:

$Mba(x) := RVb \cdot x - \frac{q \cdot x^2}{2}$

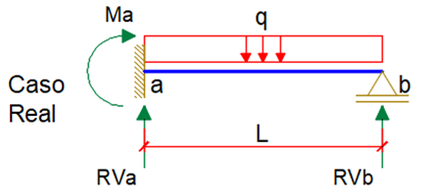
posição de valor extremo de Mba(x) $\quad \frac{dMba(x)}{dx} = 0$

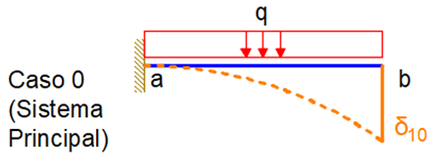
$RVb - q \cdot x = 0 \quad ---> \quad x_ext := \frac{-RVb}{-q} = 2,625\ m$

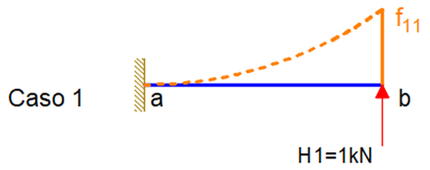
$Mba(x_ext) = 68,906\ kN\ m$ $\quad Mba(0) = 0\ kN\ m$ $\quad Mba(L) = -122,5\ kN\ m$

$Q04 := Mba(x_ext) = 68,906\ kN\ m$

Cálculo dos Deslocamentos via Método das Forças (MF) com PTV via Integração Numérica:







Aplicação do PTV:

$$\delta_{10} := \frac{\int_0^L Mba0(x)\cdot Mba0_virtual(x)\,dx}{EI}\cdot\frac{1}{H1}$$

$$Mba0_virtual(x) = Mba1(x)$$

$$\delta_{10} := \frac{\int_0^L Mba1(x)\cdot Mba0(x)\,dx}{EI}\cdot\frac{1}{H1} \qquad Mba0(x) = -\frac{q\cdot x^2}{2} \qquad Mba1(x) = H1\cdot x$$

$$\delta_{10} = \frac{1}{EI}\cdot\int_0^L (H1\cdot x)\cdot\left(-\frac{q\cdot x^2}{2}\right)dx$$

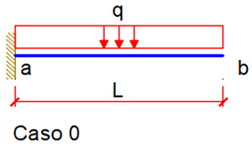
$$\delta_{10} = \frac{H1\cdot q}{2\cdot EI}\cdot\int_0^L -x^3\,dx \quad \text{---> } \delta_{10} := \frac{H1\cdot q}{2\cdot EI}\cdot\left(-\frac{L^4}{4}\right)\cdot\frac{1}{H1} = -0{,}28137\ \text{m}$$

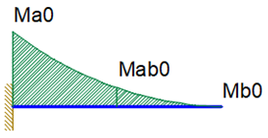
$$f_{11} := \frac{\int_0^L Mba1(x)\cdot Mba1(x)\,dx}{EI}\cdot\frac{1}{H1}$$

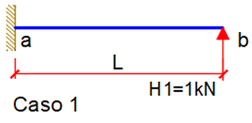
$$f_{11} = \frac{1}{EI}\cdot\int_0^L (H1\cdot x)\cdot(H1\cdot x)\,dx \quad \text{---> } f_{11} = \frac{1}{EI}\cdot\int_0^L H1^2\cdot x^2\,dx \quad \text{---> } f_{11} = \frac{H1^2}{EI}\cdot\int_0^L x^2\,dx$$

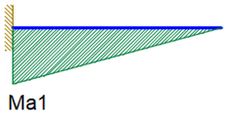
$$\text{---> } f_{11} := \frac{H1^2}{EI}\cdot\left(\frac{L^3}{3}\right)\cdot\frac{1}{H1} = 0{,}0053594\ \text{m}$$

Cálculo dos Deslocamentos via Método das Forças (MF) com PTV via Integração por Tabela de Integrais Geométricas:









Aplicação do PTV:

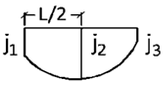
$$\delta_{10} := \frac{1}{EI}\cdot\int_0^L Mba1(x)\cdot Mba0(x)\,dx\cdot\frac{1}{H1}$$

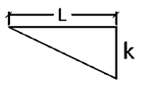
$$Ma1 := H1\cdot L = 7\ \text{kN m}$$

$$Ma0 := \frac{-q\cdot L^2}{2} = -490\ \text{kN m}$$

$$Mab0 := \frac{-q\cdot(0{,}5\cdot L)^2}{2} = -122{,}5\ \text{kN m}$$

$$Mb0 := 0$$

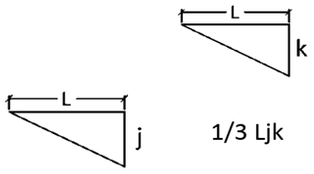




$$k := Ma1 \qquad j_2 := Mab0 \qquad j_3 := Ma0$$

$$\delta_{10} := \frac{1}{EI}\cdot\frac{1}{6}\cdot L\cdot k\cdot\left(2\cdot j_2 + j_3\right)\cdot\frac{1}{H1} = -0{,}28137\ \text{m}$$

$$f_{11} := \frac{1}{EI}\cdot\int_0^L Mba1(x)\cdot Mba1(x)\,dx\cdot\frac{1}{H1}$$



1/3 Ljk

$k := Ma1 \qquad j := Ma1$

$$f_{11} := \frac{1}{EI} \cdot \frac{1}{3} \cdot L \cdot j \cdot k \cdot \frac{1}{H1} = 0,0053594 \text{ m}$$

$Q01 = 0,28137 \text{ m}$

$Q02 = 0,0053594 \frac{\text{m}}{\text{kN}}$

$Q03 = 52,5 \text{ kN}$

$Q04 = 68,9062 \text{ kN m}$
