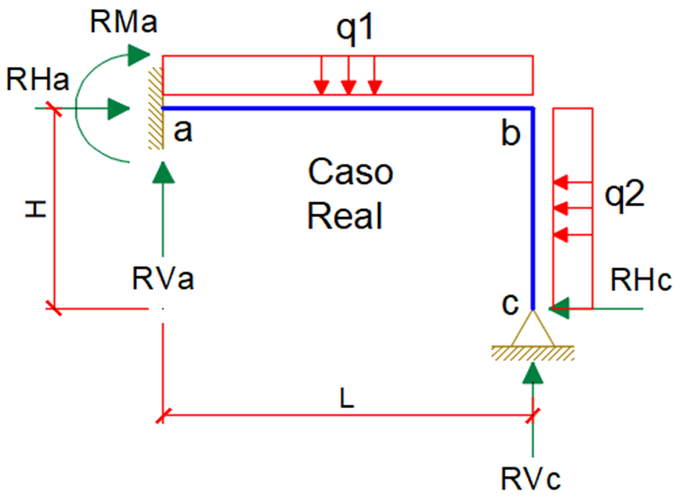
B1T4NA0910A

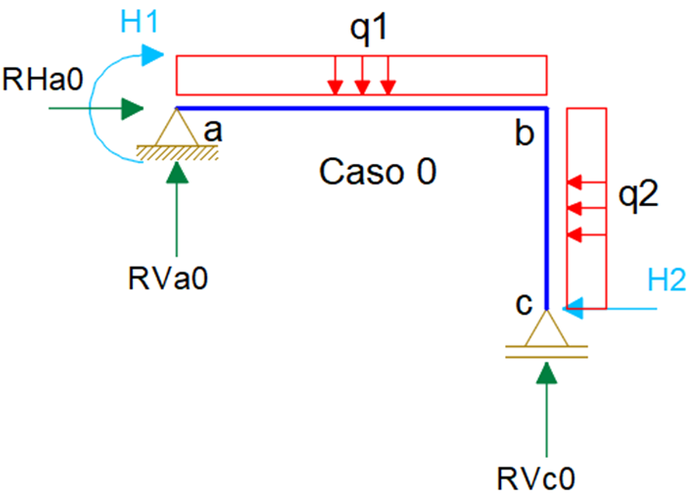
Empregando o método das forças com as redundantes hiperestáticas indicadas (H1 e H2) calcule a reação de apoio RVa respondendo os questionamentos.

* Considerar somente efeitos de flexão nos cálculos;

Para todas as barras considerar: $EI := 1\ \text{kN m}^2$

$L := 7,0\ \text{m}$ $H := 4,0\ \text{m}$ $q1 := 12\ \frac{\text{kN}}{\text{m}}$ $q2 := 20\ \frac{\text{kN}}{\text{m}}$





Q01) Qual o valor do deslocamento de rotação que ocorre no ponto "a" do caso 0 de carregamento [em rad e em módulo]?

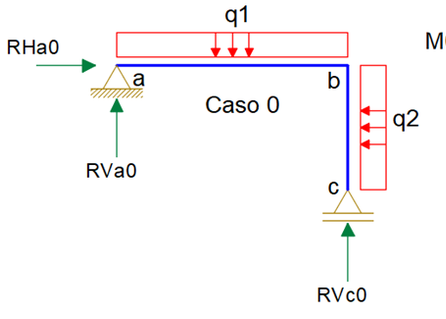
Q02) Qual o valor do deslocamento de translação que ocorre no ponto "c" do caso 0 de carregamento [em m em módulo]?

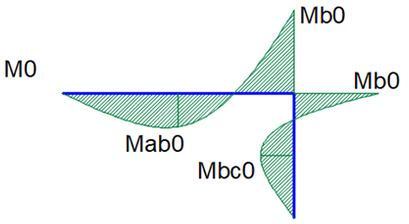
Q03) Considerando as redundantes hiperestáticas indicadas, qual o valor do coeficiente de flexibilidade "f11" [em unidades de "kN" e "m" e em módulo]?

Q04) Considerando as redundantes hiperestáticas indicadas, qual o valor do coeficiente de flexibilidade "f12" [em unidades de "kN" e "m" e em módulo]?

Q05) Qual o valor da reação de apoio RVa [em unidades de "kN" e com positivo para cima]?

Deslocamentos do Caso 0 e Coeficientes de Flexibilidade:





$\Sigma Ma = 0$ (+) antihorário

$RVc0 \cdot L - \frac{q1 \cdot L^2}{2} - \frac{q2 \cdot H^2}{2} = 0$

$RVc0 := \frac{1}{L} \cdot \left(\frac{q1 \cdot L^2}{2} + \frac{q2 \cdot H^2}{2} \right)$

$RVc0 = 64,857\ \text{kN}$

$\Sigma Fh = 0$ (+) para direita

$RHa0 := q2 \cdot H = 80\ \text{kN}$

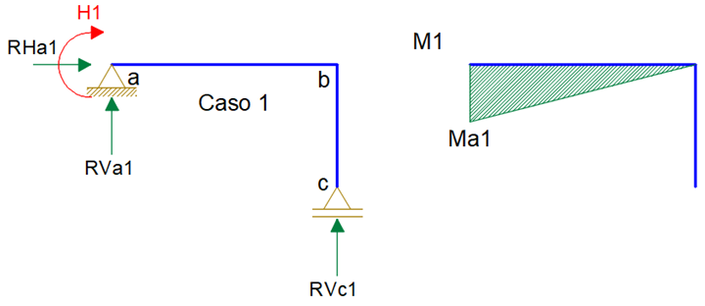
$\Sigma Fv = 0$ (+) para cima $RVa0 + RVc0 - q1 \cdot L = 0$

$RVa0 := q1 \cdot L - RVc0 = 19,143\ \text{kN}$

$Mab0 := RVa0 \cdot \frac{L}{2} - q1 \cdot \left(\frac{L}{2} \right)^2 \cdot \frac{1}{2} = -6,5\ \text{kN m}$

$Mb0 := RVa0 \cdot L - q1 \cdot L^2 \cdot \frac{1}{2} = -160\ \text{kN m}$

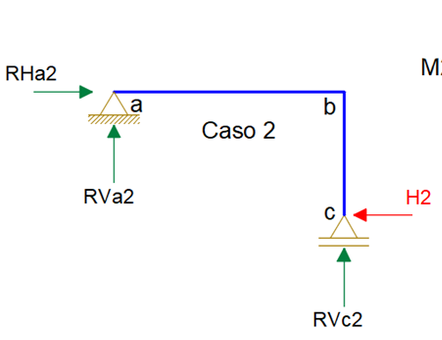
$Mbc0 := -q2 \cdot \left(\frac{H}{2}\right)^2 \cdot \frac{1}{2} = -40\ \text{kN m}$

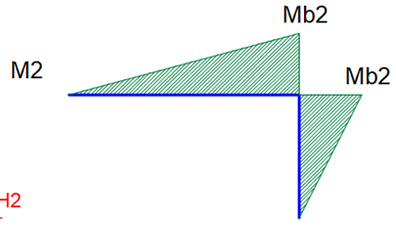


$H1 := 1\ \text{kN m}$

$RVa1 := -\frac{H1}{L} = -0,14286\ \text{kN}$

$Ma1 := 1\ \text{kN m}$





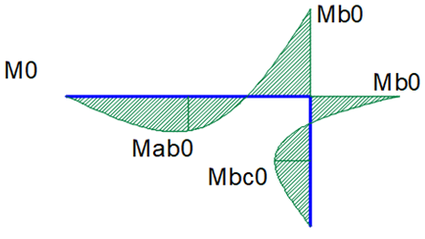
$H2 := 1\ \text{kN}$

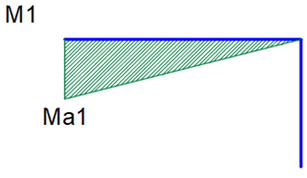
$Mb2 := -H2 \cdot H = -4\ \text{kN m}$

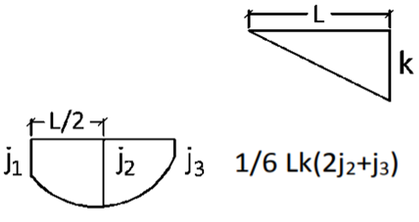
$RHa2 := H2 = 1\ \text{kN}$

$RVa2 := \frac{Mb2}{L} = -0,57143\ \text{kN}$

Cálculo de φ10:







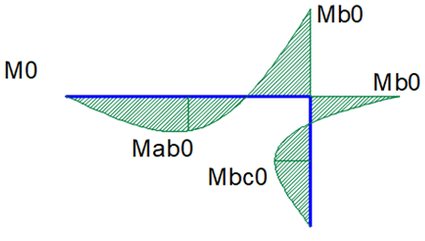
$k := Ma1 = 1\ \text{kN m}$

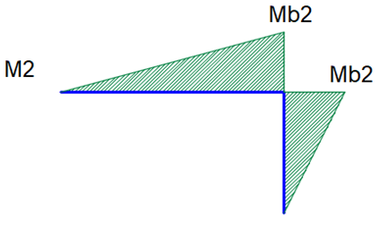
$j3 := 0 = 0\ \text{kN m}$ $\quad j2 := Mab0 = -6,5\ \text{kN m}$

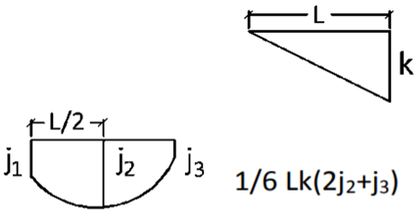
$\varphi 10 := \frac{1}{EI} \cdot \frac{1}{6} \cdot L \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{\text{kN m}} = -15,167$

$Q01 := |\varphi 10| = 15,167$

Cálculo de δ20:



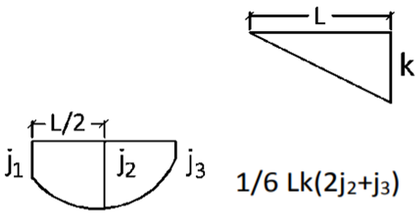




$k := Mb2 = -4\ \text{kN m}$

$j3 := Mb0 = -160\ \text{kN m}$ $\quad j2 := Mab0 = -6,5\ \text{kN m}$

$\delta 20ab := \frac{1}{EI} \cdot \frac{1}{6} \cdot L \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{\text{kN m}} = 807,3333$



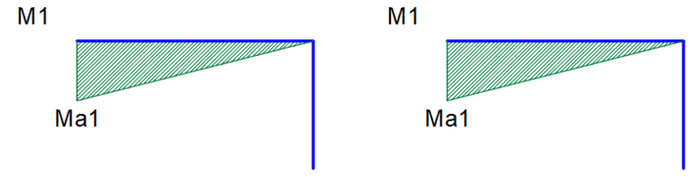
$k := Mb2 = -4\ \text{kN m}$

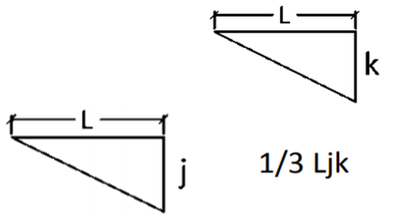
$j3 := Mb0 = -160\ \text{kN m}$ $\quad j2 := Mbc0 = -40\ \text{kN m}$

$\delta 20bc := \frac{1}{EI} \cdot \frac{1}{6} \cdot H \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{\text{kN m}} = 640$

$\delta 20 := \delta 20ab + \delta 20bc = 1447,3$ $\quad Q02 := |\delta 20| = 1447,3$

Cálculo de f11:

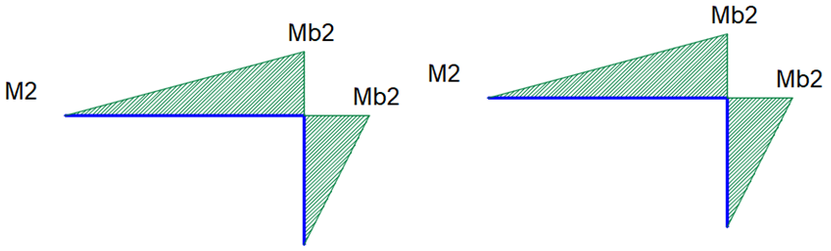


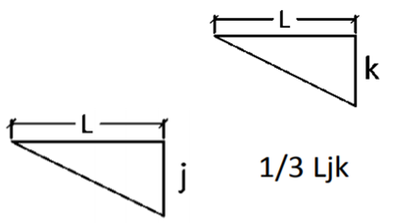


$k := Ma1 = 1\ \text{kN m}$ $\quad j := Ma1 = 1\ \text{kN m}$

$f11 := \frac{1}{EI} \cdot \frac{1}{3} \cdot L \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = 2,3333$ $\quad Q03 := |f11| = 2,3333$

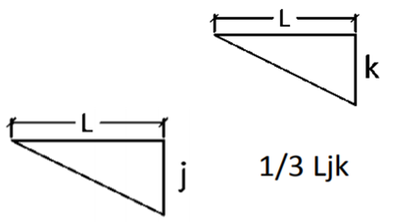
Cálculo de f22:





$k := Mb2 = -4\ \text{kN m}$ $\quad j := Mb2 = -4\ \text{kN m}$

$f22ab := \frac{1}{EI} \cdot \frac{1}{3} \cdot L \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = 37,3333$

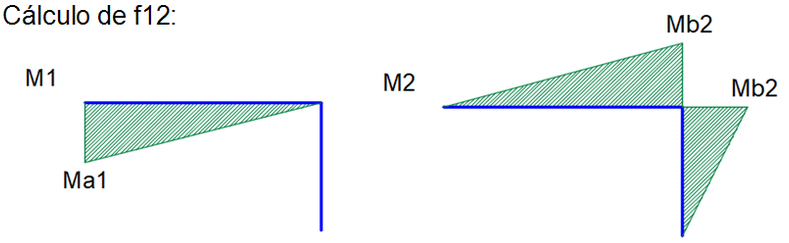


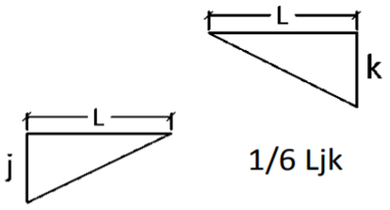
$k := Mb2 = -4\ \text{kN m}$ $\quad j := Mb2 = -4\ \text{kN m}$

$f22bc := \frac{1}{EI} \cdot \frac{1}{3} \cdot H \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = 21,3333$

$f22 := f22ab + f22bc = 58,667$

Cálculo de f12:





$k := Mb2 = -4\ \text{kN m}$ $j := Ma1 = 1\ \text{kN m}$

$f12 := \frac{1}{EI} \cdot \frac{1}{6} \cdot L \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = -4,6667$ $Q04 := |f12| = 4,6667$

Solução do Sistema de Equações:

$$A := \begin{bmatrix} f11 & f12 \\ f12 & f22 \end{bmatrix} = \begin{bmatrix} 2,3333 & -4,6667 \\ -4,6667 & 58,667 \end{bmatrix}$$

$$b := \begin{bmatrix} -\varphi 10 \\ -\delta 20 \end{bmatrix} = \begin{bmatrix} 15,167 \\ -1447,3 \end{bmatrix}$$

$R := A^{-1} \cdot b = \begin{bmatrix} -50,946 \\ -28,723 \end{bmatrix}$ $RMa := R_1 = -50,946$ $RHc := R_2 = -28,723$

Cálculo de RVa por Sobreposição de Efeitos:

$RVa := RVa0 + RVa1 \cdot RMa + RVa2 \cdot RHc = 42,834\ \text{kN}$ $Q05 := \frac{RVa}{1\ \text{kN}} = 42,834$

$Q01 = 15,167$

$Q02 = 1447,333$

$Q03 = 2,3333$

$Q04 = 4,6667$

$Q05 = 42,834$
