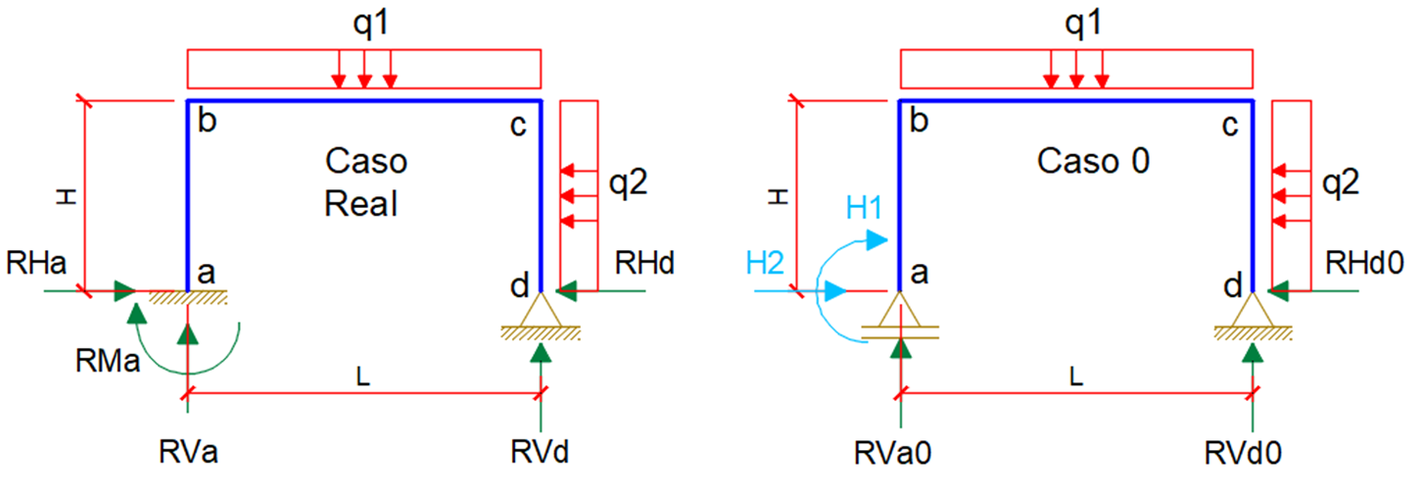
B1T4MA0914A

Empregando o método das forças com as redundantes hiperestáticas indicadas (H1 e H2) calcule a reação de apoio RVa respondendo os questionamentos.

* Considerar somente efeitos de flexão nos cálculos;

Para todas as barras considerar: $EI := 1\ \text{kN m}^2$

$L := 7,0\ \text{m}$ $H := 4,0\ \text{m}$ $q1 := 10\ \frac{\text{kN}}{\text{m}}$ $q2 := 20\ \frac{\text{kN}}{\text{m}}$



Q01) Qual o valor do deslocamento de rotação que ocorre no ponto "a" do caso 0 de carregamento [em rad e em módulo]?

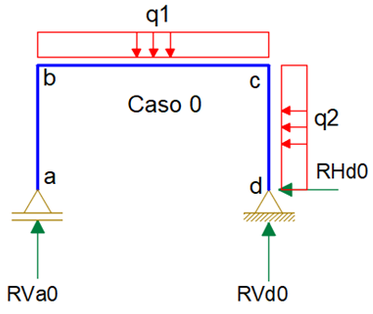
Q02) Qual o valor do deslocamento de translação horizontal que ocorre no ponto "a" do caso 0 de carregamento [em m em módulo]?

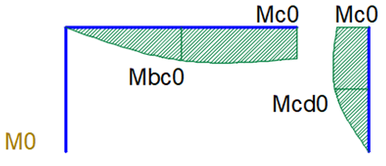
Q03) Considerando as redundantes hiperestáticas indicadas, qual o valor do coeficiente de flexibilidade "f11" [em unidades de "kN" e "m" e em módulo]?

Q04) Considerando as redundantes hiperestáticas indicadas, qual o valor do coeficiente de flexibilidade "f12" [em unidades de "kN" e "m" e em módulo]?

Q05) Qual o valor da reação de apoio RVa [em unidades de "kN" e com positivo para cima]?

Deslocamentos do Caso 0 e Coeficientes de Flexibilidade:





$\Sigma Ma = 0$ (+) antihorário

$$RVa0 \cdot L - \frac{q1 \cdot L^2}{2} - \frac{q2 \cdot H^2}{2} = 0$$

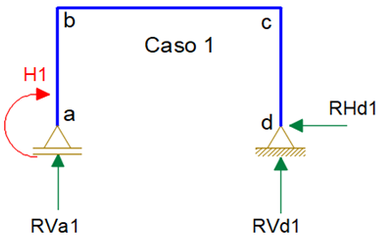
$$RVa0 := \frac{1}{L} \cdot \left(\frac{q1 \cdot L^2}{2} + \frac{q2 \cdot H^2}{2} \right)$$

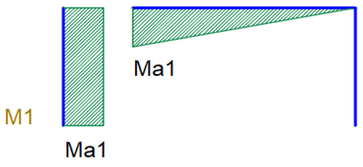
$RVa0 = 57,857\ \text{kN}$

$$Mbc0 := RVa0 \cdot \frac{L}{2} - q1 \cdot \left(\frac{L}{2} \right)^2 \cdot \frac{1}{2} = 141,25\ \text{kN m}$$

$$Mc0 := RVa0 \cdot L - q1 \cdot L^2 \cdot \frac{1}{2} = 160\ \text{kN m}$$

$$Mcd0 := RVa0 \cdot L - q1 \cdot L^2 \cdot \frac{1}{2} - q2 \cdot \left(\frac{H}{2} \right)^2 \cdot \frac{1}{2} = 120\ \text{kN m}$$

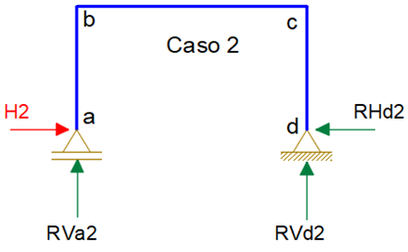


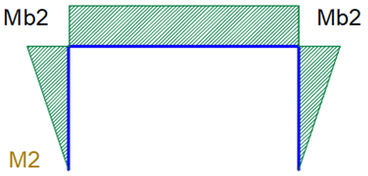


$H1 := 1\ \text{kN m}$

$RVa1 := -\frac{H1}{L} = -0,14286\ \text{kN}$

$Ma1 := 1\ \text{kN m}$





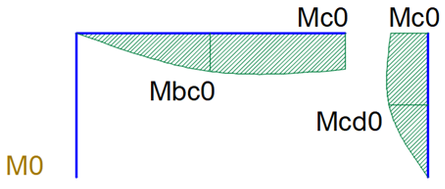
$H2 := 1\ \text{kN}$

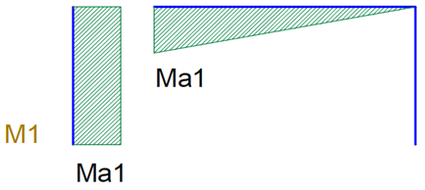
$Mb2 := -H2 \cdot H = -4\ \text{kN m}$

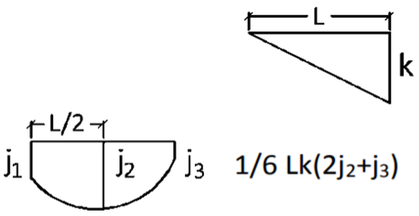
$RHa2 := H2 = 1\ \text{kN}$

$RVa2 := 0$

Cálculo de φ10:







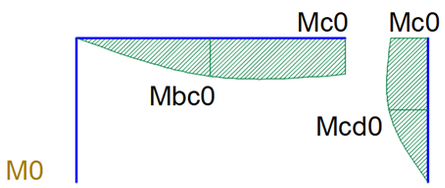
$k := Ma1 = 1\ \text{kN m}$

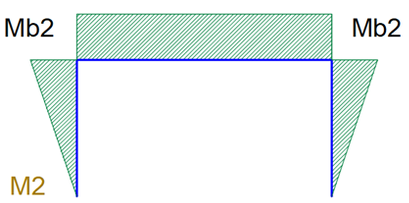
$j3 := 0 = 0\ \text{kN m}$ $\quad j2 := Mbc0 = 141,25\ \text{kN m}$

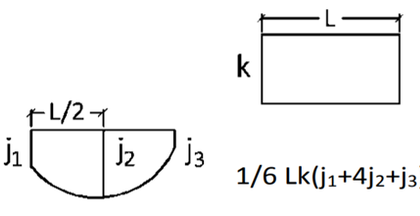
$\varphi 10 := \frac{1}{EI} \cdot \frac{1}{6} \cdot L \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{\text{kN m}} = 329,58$

$Q01 := |\varphi 10| = 329,58$

Cálculo de δ20:



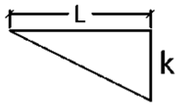




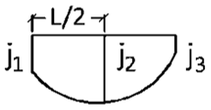
$k := Mb2 = -4\ \text{kN m}$ $\quad j1 := 0$

$j2 := Mbc0 = 141,25\ \text{kN m}$ $\quad j3 := Mc0 = 160\ \text{kN m}$

$\delta 20bc := \frac{1}{EI} \cdot \frac{1}{6} \cdot L \cdot k \cdot (j1 + 4 \cdot j2 + j3) \cdot \frac{1}{\text{kN m}} = -3383,3333$



$k := Mb2 = -4\ \text{kN m}$



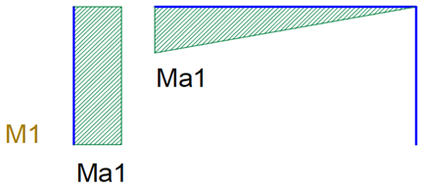
1/6 Lk(2j₂+j₃)

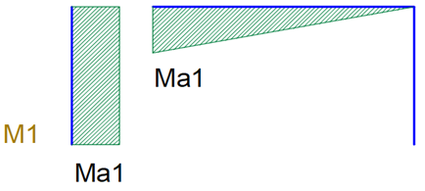
$j2 := Mcd0 = 120\ \text{kN m}$ $j3 := Mc0 = 160\ \text{kN m}$

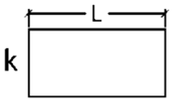
$\delta 20cd := \frac{1}{EI} \cdot \frac{1}{6} \cdot H \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{\text{kN m}} = -1066,6667$

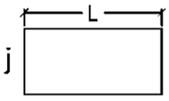
$\delta 20 := \delta 20bc + \delta 20cd = -4450$ $Q02 := |\delta 20| = 4450$

Cálculo de f11:





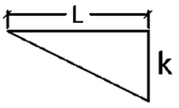


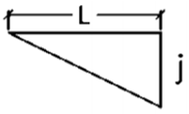


Ljk

$k := Ma1 = 1\ \text{kN m}$ $j := Ma1 = 1\ \text{kN m}$

$f11ab := \frac{1}{EI} \cdot H \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = 4$





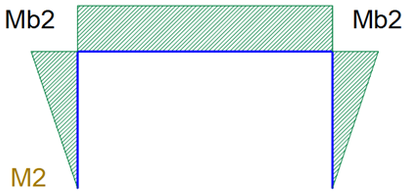
1/3 Ljk

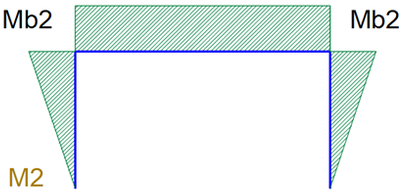
$k := Ma1 = 1\ \text{kN m}$ $j := Ma1 = 1\ \text{kN m}$

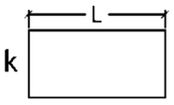
$f11bc := \frac{1}{EI} \cdot \frac{1}{3} \cdot L \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = 2,3333$

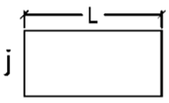
$f11 := f11ab + f11bc = 6,3333$ $Q03 := |f11| = 6,3333$

Cálculo de f22:





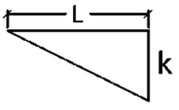


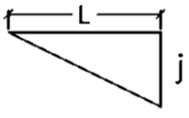


Ljk

$k := Mb2 = -4\ \text{kN m}$ $j := Mb2 = -4\ \text{kN m}$

$f22bc := \frac{1}{EI} \cdot L \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = 112$





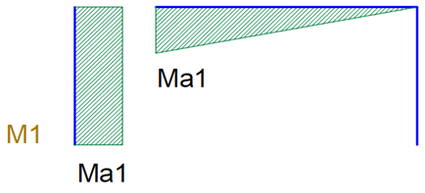
1/3 Ljk

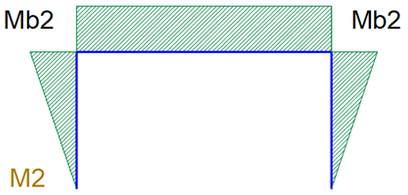
$k := Mb2 = -4\ \text{kN m}$ $j := Mb2 = -4\ \text{kN m}$

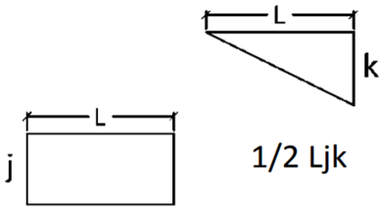
$f22ab := \frac{1}{EI} \cdot \frac{1}{3} \cdot H \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = 21,3333$

$f22 := 2 \cdot f22ab + f22bc = 154,67$

Cálculo de f12:

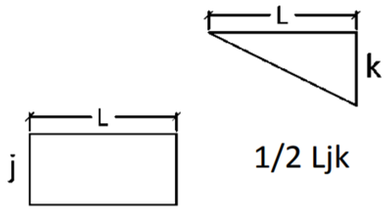






$k := Mb2 = -4\ \text{kN m}$ $j := Ma1 = 1\ \text{kN m}$

$f12ab := \frac{1}{EI} \cdot \frac{1}{2} \cdot H \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = -8$



$k := Ma1 = 1\ \text{kN m}$ $j := Mb2 = -4\ \text{kN m}$

$f12bc := \frac{1}{EI} \cdot \frac{1}{2} \cdot L \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = -14$

$f12 := f12ab + f12bc = -22$ $Q04 := |f12| = 22$

Solução do Sistema de Equações:

$A := \begin{bmatrix} f11 & f12 \\ f12 & f22 \end{bmatrix} = \begin{bmatrix} 6,3333 & -22 \\ -22 & 154,67 \end{bmatrix}$

$b := \begin{bmatrix} -\varphi 10 \\ -\delta 20 \end{bmatrix} = \begin{bmatrix} -329,58 \\ 4450 \end{bmatrix}$

$R := A^{-1} \cdot b = \begin{bmatrix} 94,691 \\ 42,24 \end{bmatrix}$ $RMa := R_1 = 94,691$ $RHc := R_2 = 42,2405$

Cálculo de RVa por Sobreposição de Efeitos:

$RVa := RVa0 + RVa1 \cdot RMa + RVa2 \cdot RHc = 44,33\ \text{kN}$ $Q05 := \frac{RVa}{1\ \text{kN}} = 44,3299$

$Q01 = 329,58$

$Q02 = 4450$

$Q03 = 6,3333$

$Q04 = 22$

$Q05 = 44,3299$
