B1T4NA0903A

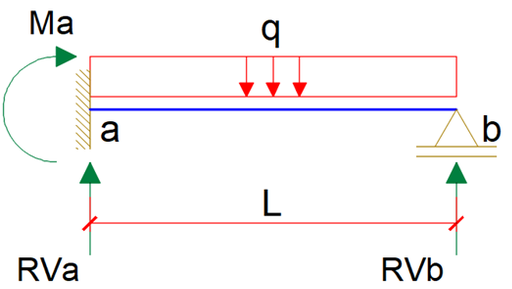
Empregando o método das forças calcule o máximo momento fletor que ocorre na viga respondendo os questionamentos.

* Adotar obrigatoriamente a reação de apoio vertical no ponto "b" como redudante hiperestática.

Q01) Qual o valor do deslocamento vertical que ocorre no ponto "b" do caso 0 de carregamento [em m e em módulo]?
Q02) Qual o valor do deslocamento vertical que ocorre no ponto "b" (coeficiente de flexibilidade) do caso 1 de carregamento [em m/kN e em módulo]?
Q03) Qual o valor da reação vertical no ponto "b" do caso real [kN positivo para cima]?
Q04) Qual o máximo momento fletor que ocorre na viga [em kNm e conforme a convenção de sinais]?

$q := 30,0\ \frac{kN}{m}$ $L := 8,0\ m$

$E := 15\ GPa$

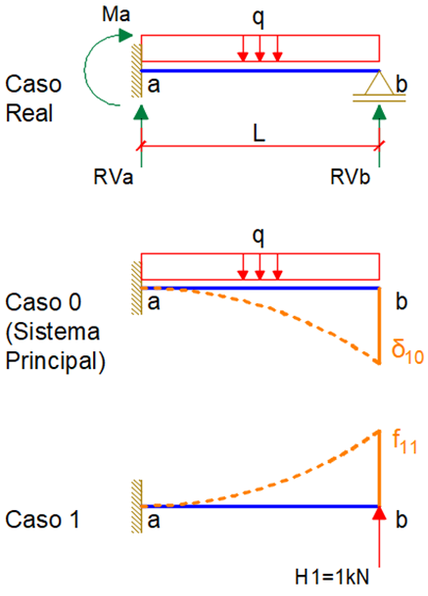


Dimensões da seção transversal retangular:

$b := 25\ cm$ $h := 60\ cm$

** Considerar nos cálculos somente efeitos de flexão;

Solução via Método das Forças (MF) com Integração da Linha Elástica:



$$EI := \frac{E \cdot b \cdot h^3}{12} = 67500\ kN\ m^2$$

Condição e Compatibilidade: $\delta_{10} + RVb \cdot f_{11} = 0$

Caso 0:

$$Mba0(x) = -\frac{q \cdot x^2}{2}$$

$$\theta ba0(x) = \frac{1}{EI} \cdot \left(\int Mba(x)\,dx + \theta b \right) \text{ ---> } \theta ba0(x) = \frac{1}{EI} \cdot \left(-\frac{q \cdot x^3}{6} + \theta b \right)$$

$$\theta ba0(L) = 0 \text{ ---> } 0 = \frac{1}{EI} \cdot \left(-\frac{q \cdot L^3}{6} + \theta b0 \right) \text{ ---> } \theta b0 := \frac{q \cdot L^3}{6}$$

$$\delta ba0(x) = \int \theta ba0(x)\,dx + \delta b0$$

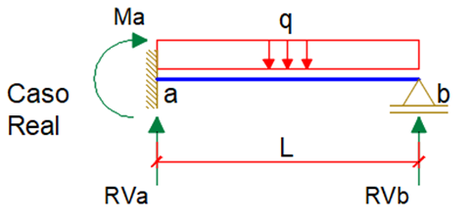
$$\delta ba0(x) = \frac{1}{EI} \cdot \left(-\frac{q \cdot x^4}{24} + \theta b0 \cdot x + \delta b0 \right)$$

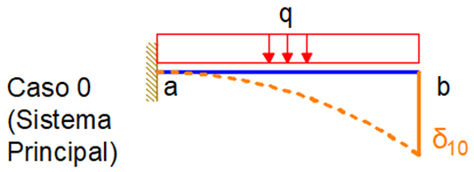
$$\delta ba0(L) = 0 \text{ ---> } 0 = \frac{1}{EI} \cdot \left(-\frac{q \cdot L^4}{24} + \frac{q \cdot L^3}{6} \cdot L + \delta b0 \right)$$

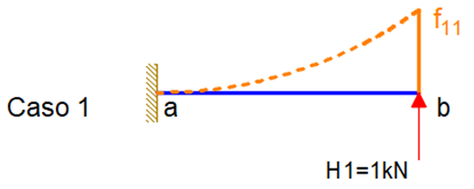
$$\delta b0 = \frac{q \cdot L^4}{24} - \frac{q \cdot L^4}{6} \text{ ---> } \delta b0 = q \cdot L^4 \cdot \left(\frac{1}{24} - \frac{1}{6} \right) \text{ ---> } \delta b0 := -\frac{q \cdot L^4}{8}$$

$$\delta ba0(x) := \frac{1}{EI} \cdot \left(-\frac{q \cdot x^4}{24} + \frac{q \cdot L^3}{6} \cdot x - \frac{q \cdot L^4}{8} \right)$$

$\delta_{10} := \delta ba0(0) = -0,22756\ \text{m}$ $\quad \frac{\delta b0}{EI} = -0,22756\ \text{m}$ $\quad Q01 := \left|\delta_{10}\right| = 0,22756\ \text{m}$







Caso 1: $H1 := 1\ \text{kN}$

$Mba1(x) = H1 \cdot x$

$\theta ba1(x) = \frac{1}{EI} \cdot \left(\int Mba1(x)\,\mathrm{d}\,x + \theta b1 \right)$

$\theta ba1(x) = \frac{1}{EI} \cdot \left(\frac{H1 \cdot x^2}{2} + \theta b1 \right)$

$\theta ba1(L) = 0 \quad \text{---->} \quad \theta b1 = -\frac{H1 \cdot L^2}{2}$

$\delta ba1(x) = \int \theta ba1(x)\,\mathrm{d}\,x + \delta b1$

$\delta ba1(x) = \frac{1}{EI} \cdot \left(\frac{H1 \cdot x^3}{6} + \theta b1 \cdot x + \delta b1 \right)$

$\delta ba1(L) = 0 \quad \text{---->} \quad 0 = \frac{1}{EI} \cdot \left(\frac{H1 \cdot L^3}{6} - \frac{H1 \cdot L^2}{2} \cdot L + \delta b1 \right)$

$\delta b1 = -\frac{H1 \cdot L^3}{6} + \frac{H1 \cdot L^3}{2} \quad \text{---->} \quad \delta b1 = H1 \cdot L^3 \cdot \left(-\frac{1}{6} + \frac{1}{2} \right)$

$\delta b1 := \frac{H1 \cdot L^3}{3}$

$\delta ba1(x) := \frac{1}{EI} \cdot \left(\frac{H1 \cdot x^3}{6} - \frac{H1 \cdot L^2}{2} \cdot x + \frac{H1 \cdot L^3}{3} \right)$

$f_{11} := \frac{\delta ba1(0)}{1\ \text{kN}} = 0,0025284\ \frac{\text{m}}{\text{kN}}$ $\quad \frac{H1 \cdot L^3}{3 \cdot EI} = 0,0025284\ \text{m}$ $\quad Q02 := \left|f_{11}\right| = 0,0025284\ \frac{\text{m}}{\text{kN}}$

Condição e Compatibilidade: $\delta_{10} + RVb \cdot f_{11} = 0 \quad \text{---->} \quad RVb := \frac{-\delta_{10}}{f_{11}} = 90\ \text{kN}$ $\quad Q03 := RVb = 90\ \text{kN}$

Equação de momentos fletores do caso real:

$Mba(x) := RVb \cdot x - \frac{q \cdot x^2}{2}$

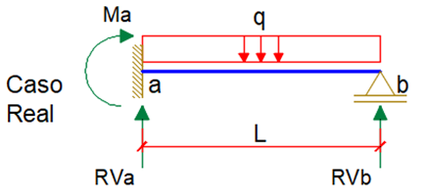
posição de valor extremo de Mba(x): $\frac{dMba(x)}{dx} = 0$

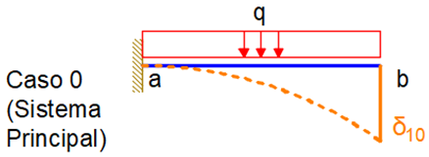
$RVb - q \cdot x = 0 \quad \text{---->} \quad x_ext := \frac{-RVb}{-q} = 3\ \text{m}$

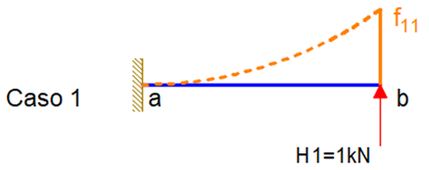
$Mba(x_ext) = 135\ \text{kN m}$ $\quad Mba(0) = 0\ \text{kN m}$ $\quad Mba(L) = -240\ \text{kN m}$

$Q04 := Mba(x_ext) = 135\ \text{kN m}$

Cálculo dos Deslocamentos via Método das Forças (MF) com PTV via Integração Numérica:







Aplicação do PTV:

$$\delta_{10} := \frac{\int_0^L Mba0(x)\cdot Mba0_virtual(x)\,dx}{EI}\cdot\frac{1}{H1}$$

$$Mba0_virtual(x) = Mba1(x)$$

$$\delta_{10} := \frac{\int_0^L Mba1(x)\cdot Mba0(x)\,dx}{EI}\cdot\frac{1}{H1} \qquad Mba0(x) = -\frac{q\cdot x^2}{2} \qquad Mba1(x) = H1\cdot x$$

$$\delta_{10} = \frac{1}{EI}\cdot\int_0^L (H1\cdot x)\cdot\left(-\frac{q\cdot x^2}{2}\right)dx$$

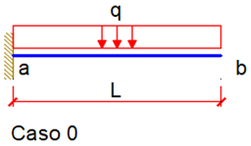
$$\delta_{10} = \frac{H1\cdot q}{2\cdot EI}\cdot\int_0^L -x^3\,dx \quad \text{---> } \delta_{10} := \frac{H1\cdot q}{2\cdot EI}\cdot\left(-\frac{L^4}{4}\right)\cdot\frac{1}{H1} = -0{,}22756\ \text{m}$$

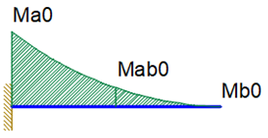
$$f_{11} := \frac{\int_0^L Mba1(x)\cdot Mba1(x)\,dx}{EI}\cdot\frac{1}{H1}$$

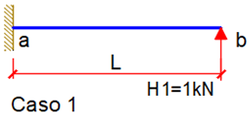
$$f_{11} = \frac{1}{EI}\cdot\int_0^L (H1\cdot x)\cdot(H1\cdot x)\,dx \quad \text{---> } f_{11} = \frac{1}{EI}\cdot\int_0^L H1^2\cdot x^2\,dx \quad \text{---> } f_{11} = \frac{H1^2}{EI}\cdot\int_0^L x^2\,dx$$

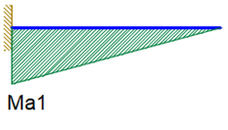
$$\text{---> } f_{11} := \frac{H1^2}{EI}\cdot\left(\frac{L^3}{3}\right)\cdot\frac{1}{H1^2} = 0{,}0025284\ \frac{\text{m}}{\text{kN}}$$

Cálculo dos Deslocamentos via Método das Forças (MF) com PTV via Integração por Tabela de Integrais Geométricas:









Aplicação do PTV:

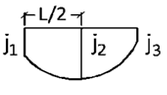
$$\delta_{10} := \frac{1}{EI}\cdot\int_0^L Mba1(x)\cdot Mba0(x)\,dx\cdot\frac{1}{H1}$$

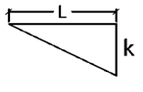
$$Ma1 := H1\cdot L = 8\ \text{kN m}$$

$$Ma0 := \frac{-q\cdot L^2}{2} = -960\ \text{kN m}$$

$$Mab0 := \frac{-q\cdot(0{,}5\cdot L)^2}{2} = -240\ \text{kN m}$$

$$Mb0 := 0$$



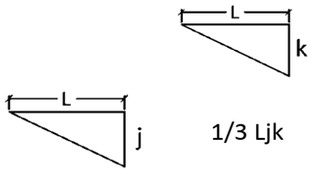


1/6 Lk(2j2+j3)

$$k := Ma1 \qquad j_2 := Mab0 \qquad j_3 := Ma0$$

$$\delta_{10} := \frac{1}{EI}\cdot\frac{1}{6}\cdot L\cdot k\cdot\left(2\cdot j_2 + j_3\right)\cdot\frac{1}{H1} = -0{,}22756\ \text{m}$$

$$f_{11} := \frac{1}{EI}\cdot\int_0^L Mba1(x)\cdot Mba1(x)\,dx\cdot\frac{1}{H1}$$



1/3 Ljk

$k := Ma1 \qquad j := Ma1$

$$f_{11} := \frac{1}{EI} \cdot \frac{1}{3} \cdot L \cdot j \cdot k \cdot \frac{1}{H1^{2}} = 0,0025284 \ \frac{m}{kN}$$

$Q01 = 0,22756 \ m$

$Q02 = 0,0025284 \ \frac{m}{kN}$

$Q03 = 90 \ kN$

$Q04 = 135 \ kN \ m$
