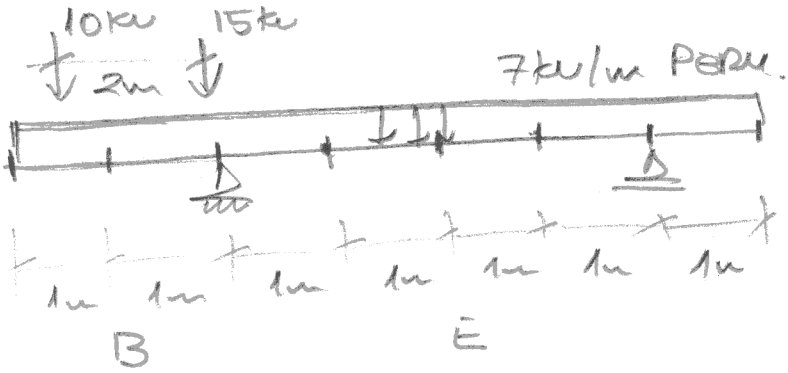
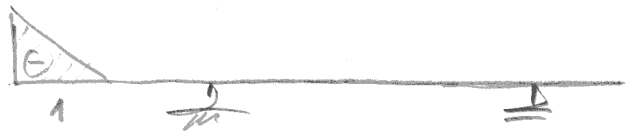
TE15 – B3 – 1000

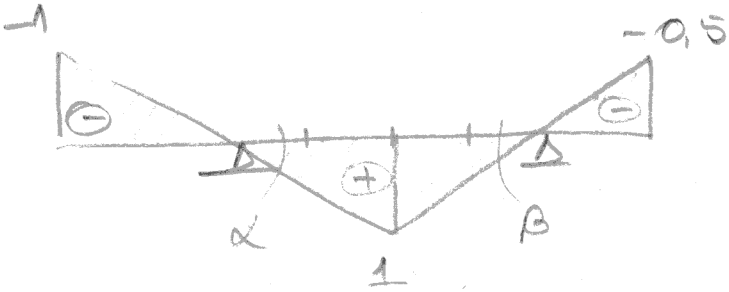


Para B: 

$M_{B\,MÁX} = 7\,kN/m \times (1 \times -1)/2 = \underline{-3{,}5\,kNm}$

$M_{B\,MÍN} = 7\,kN/m \times \frac{1 \times (-1)}{2} + (-1) \times 15\,kN = \underline{-18{,}5\,kNm}$

Para E:



$tg\,\alpha = tg\,\beta = 0{,}5$

$A_{\oplus} = 4 \times 1/2 = +2\,m^2$

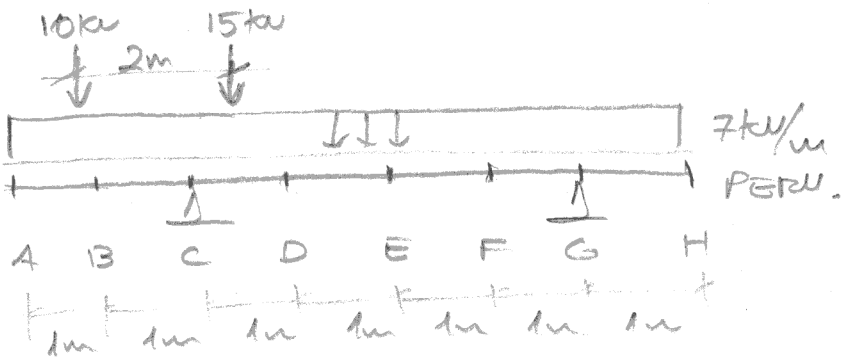
$A_{\ominus} = -1 \times 3/2 - 0{,}5 \times 1/2 = -1{,}25\,m^2$

$M_{E\,MÁX} = 7\,kN/m\,(2 - 1{,}25)\,m^2 + 15\,kN \times (+1m) = \underline{20{,}25\,kNm}$

$M_{E\,MÍN} = 7\,kN/m\,(2 - 1{,}25)\,m^2 + 15\,kN \times (-1m) = \underline{-9{,}75\,kNm}$

(kNm)	$M_{MÍN.\,TOT.}$	$M_{MÁX.\,TOT.}$
B	-18,50	-3,50
E	-9,73	+20,25

TE15 - B3 - 1001



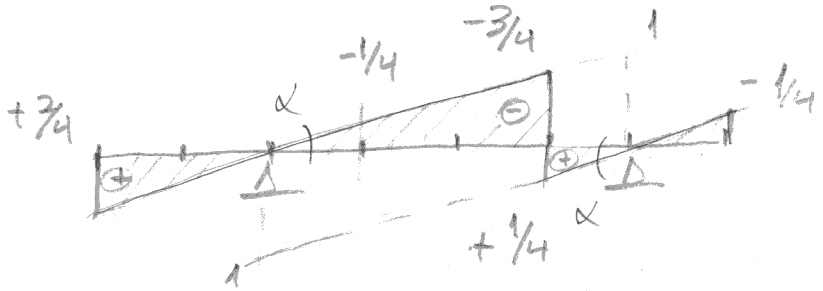
Paso B:



$Q_{B\,MÍN} = (1m) \times (-1) \times 7\,kN/m + 15\,kN \times (-1) = -22{,}00\,kN$

$Q_{B\,MÁX} = (1m) \times (-1) \times 7\,kN/m = -7{,}00\,kN$

Paso F



$tg\,\alpha = 1/4$

$A\oplus = \left(\frac{3}{4} \times 2 \times \frac{1}{2}\right) + \left(\frac{1}{4} \times 1 \times \frac{1}{2}\right) = 0{,}625\,m$ ⊕

$A\ominus = \left(-\frac{3}{4} \times 3 \times \frac{1}{2}\right) + \left(-\frac{1}{4} \times 1 \times \frac{1}{2}\right) = -1{,}25\,m$ ⊖

$Q_{F\,MÍN} = 7\frac{kN}{m}(0{,}625 - 1{,}25)\,m + 15\,kN \times \left(-\frac{3}{4}\right) + 10\,kN \times \left(-\frac{1}{4}\right)$

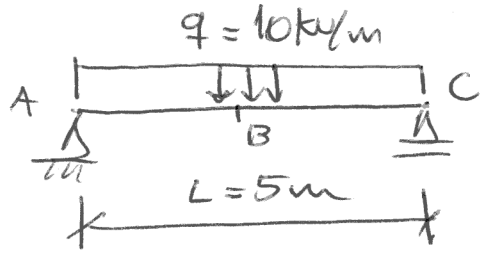
$= -18{,}125\,kN$

$Q_{F\,MÁX} = 7\frac{kN}{m}(0{,}625 - 1{,}25)\,m + 15\,kN \times \left(+\frac{3}{4}\right) = +3{,}125\,kN$

(kN)	$Q_{MÍN\,TOT.}$	$Q_{MÁX.\,TOT.}$
B	-22,000	-7,000
F	-18,125	+3,125

TE15 - B3 - 2000

DEFLEXÃO DO PONTO B DA VIGA DE SEÇÃO RETANGULAR



$E = 20 \times 10^6\ kN/m^2$

$b = 0{,}20m \quad h = 0{,}50m$

* B NO MEIO DO VÃO.

$$EI = 20\times10^6\ kN/m^2 \times 0{,}20m \times (0{,}50m)^3/12 = \frac{125000}{3}\ kN m^2$$

$$V_A = V_C = 10\ kN/m \times (5m)/2 = 25\ kN$$

$$M_{AB}(x) = 25x - 10x^2/2$$

$$\phi_{AB}(x) = \frac{1}{EI}\int 25x - 10\frac{x^2}{2}\,dx = \frac{1}{EI}\left(25\frac{x^2}{2} - 10\frac{x^3}{6} + C_1\right)$$

$$\delta_{AB}(x) = \frac{1}{EI}\int \frac{25x^2}{2} - \frac{10x^3}{6} + C_1\,dx = \frac{1}{EI}\left(\frac{25x^3}{6} - \frac{10x^4}{24} + C_1x + C_2\right)$$

$$\delta_{AB}(x=0) = 0 \qquad \delta_{AB}(x=5) = 0$$

Logo: $\frac{25.0^3}{6} - \frac{10.0^4}{24} + C_1.0 + C_2 = 0 \rightsquigarrow C_2 = 0$

$$\frac{25.5^3}{6} - \frac{10.5^4}{24} + C_1.5 = 0 \rightsquigarrow C_1 = -\frac{625}{12}$$

$$\delta_{AB}(x) = \frac{1}{EI}\left(\frac{25}{6}x^3 - \frac{10}{24}x^4 - \frac{625}{12}x\right)$$

$$\delta_{AB}(x=2{,}5m) = \frac{1}{EI}.\frac{-15625}{192} = \frac{3}{125000} \times \frac{(-15625)}{192}$$

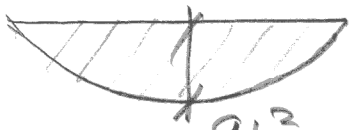
$$\delta_{AB}(x=2{,}5m) = \frac{-1}{512} = -0{,}001953\ m \downarrow$$

DAS TABELAS DE AÇO/MADEIRA: $\delta = \frac{5qL^4}{384EI}$

$$\delta = \frac{5\times10\times3^4\times3}{384\times125000} = \frac{1}{512}$$ ok! //

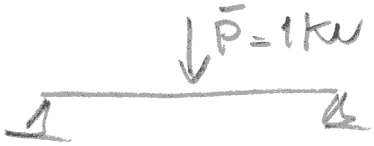
APLICANDO O PRINCÍPIO DOS TRABALHOS VIRTUAIS:

(M) REAL

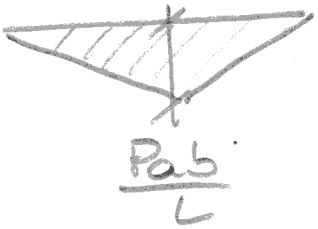


$$\frac{qL^2}{8} = \frac{10 \times 5^2}{8} = \frac{125}{4}$$

CARREG. VIRTUAL



$(\bar{M})$ VIRTUAL



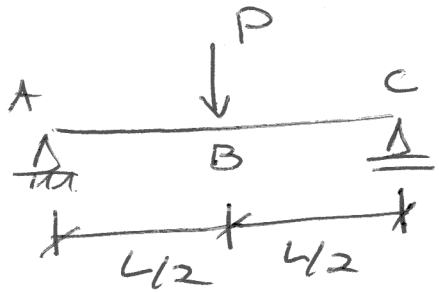
$$\frac{Pab}{L} = \frac{1 \times 2{,}5 \times 2{,}5}{5} = 5/4$$

$$\delta_B = \frac{1}{EI} \int_0^5 \bar{M} M \, dx = \frac{1}{EI} \times \frac{1}{6} 5 \times \frac{5}{4} \left[0 + 2 \times \frac{125}{4} + 0 \right.$$

$$\left. - 0{,}5 \times 0{,}5 \left(0 - 2 \times \frac{125}{4} + 0 \right) \right]$$

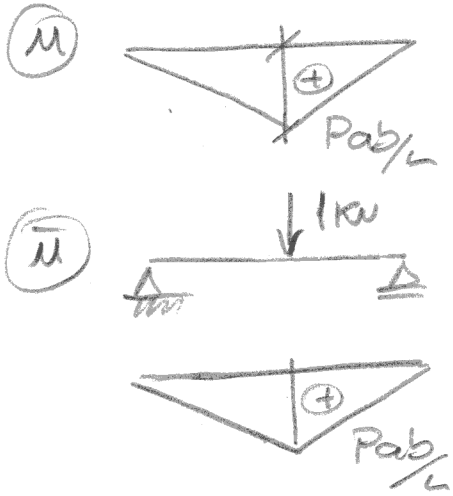
$$\delta_B = \frac{1}{EI} 81{,}380 = \frac{81{,}380}{20 \times 10^6 \times 0{,}2 \times 0{,}5^3/12} = 0{,}001953 \text{ mokl}$$

TE15 - B3 - 2001



EI

CALCULAR A DEFLEXÃO DO PONTO B CONSIDERANDO SOMENTE EFEITOS DE FLEXÃO.



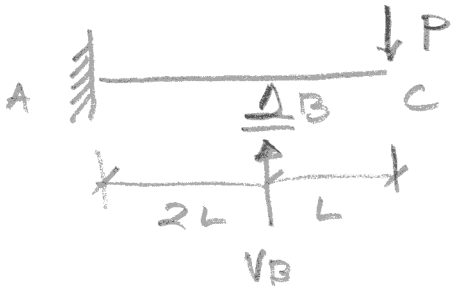
$$P \times L/2 \times L/2 \times 1/L = \frac{PL}{4}$$

$$1 \times L/2 \times L/2 \times 1/L = L/4$$

$$\int \bar{M} M \, dx = 2 \times 1/3 \, Ljk = \frac{2}{3} \cdot \frac{L}{2} \cdot \frac{PL}{4} \cdot \frac{L}{4} = \boxed{\frac{PL^3}{48}}$$

TE15 - B3 - 3000

- CALCULAR A REAÇÃO DE APOIO VB:

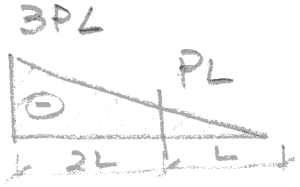


* CONSIDERAR SOMENTE EFEITOS DE FLEXÃO
* EI p/ TODAS AS BARRAS

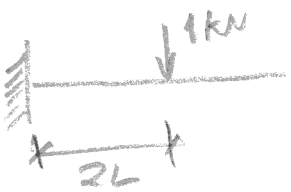
CASO 0



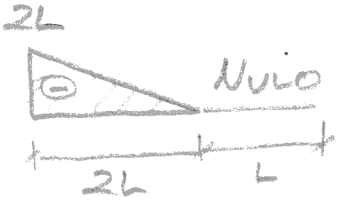
M_0



CASO 1



M_1



$$\delta_{10} = \frac{1}{EI}\frac{1}{6} L_j (2k_1 + k_2) = \frac{1}{EI}\frac{1}{6} 2L(-2L)\left(2(-3PL) + (-PL)\right)$$

$$= \frac{1}{6EI}(-4L^2)(-6PL - PL) = -\frac{4L^2}{6EI}(-7PL) = \frac{14}{3} PL^3$$

$$f_{11} = \frac{1}{EI}\frac{1}{3} L_j k = \frac{1}{3EI} 2L \cdot 2L \cdot 2L = \frac{8L^3}{3EI}$$

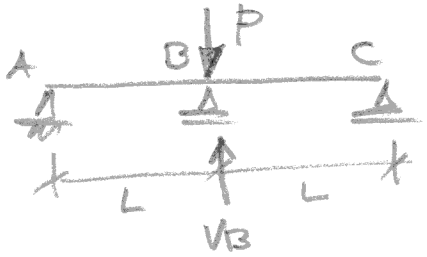
Eq. DE COMPATIBILIDADE: $\delta_{10} + f_{11} V_B = 0$

$$V_B = -\delta_{10}/f_{11} = \frac{-14/3\, PL^3 \times 1/EI}{8/3\, L^3 \times 1/EI} = -\frac{7}{4} P$$

RESPOSTA: $\boxed{V_B = \frac{7}{4} P \uparrow}$

TE15 - B3 - 3001

CALCULAR A REAÇÃO DE APOIO V_B.



* TODAS AS BARRAS POSSUEM O MESMO EI;
* CONSIDERAR EFEITOS DE FLEXÃO APENAS;

CASO 0



M_0



$\frac{PL \cdot L}{2L} = \frac{PL}{2}$

CASO 1



M_1



$L/2$

$$\delta_{10} = \frac{1}{EI} 2 \times \frac{1}{3} L \times \frac{PL}{2} \times \frac{1}{2} = \frac{1}{6EI} \times PL^3$$

$$f_{11} = \frac{1}{EI} 2 \times \frac{1}{3} L \times 1 \times \frac{1}{2} = \frac{1}{6EI} \times L^3$$

EQ DE COMPATIBILIDADE: $\delta_{10} + f_{11} V_B = 0$

$$V_B = -\delta_{10} / f_{11} = -\frac{\frac{1}{6EI} \times PL^3}{\frac{1}{6EI} \times L^3} = -P$$

OU SEJA $\boxed{V_B = P \uparrow}$