**TE16-AB-B1a**

## Problema A

*restart*

**A01:**
- Rigidez é uma relação entre **força e deslocamento;**

**A02:**

$d := 1.00$ :
$L := 0.95$ :
$E := 1.0e5$ :
$fy := 210.0$ :
$\varepsilon := 0.0010$ :

$$A := \frac{\mathrm{Pi} \cdot (0.01 \cdot d)^2}{4} :$$

**(1)**

$$k := \frac{1000 \cdot E \cdot A}{L} :$$

$N := 1000 \cdot E \cdot A \cdot \varepsilon$ :
$\delta := \varepsilon \cdot L$ :
$Ny := A \cdot 1000 \cdot fy \cdot 0.1$ :

$$print\left( \text{"k="}, k, \frac{kN}{m} \right)$$

$$\text{"k="}, 8267.349089, \frac{kN}{m} \quad \textbf{(2)}$$

$$print(\text{"N="}, N, kN)$$

$$\text{"N="}, 7.853981635, kN \quad \textbf{(3)}$$

$$print(\text{"δ="}, \delta, m)$$

$$\text{"δ="}, 0.000950, m$$

$$print(\text{"Ny="}, Ny, tf)$$

$$\text{"Ny="}, 1.649336143, tf \quad \textbf{(5)}$$

**A03:**
- Modelo de pórtico;
- 3 rótulas com 2 barras;
- 0 rótulas com 4 barras;
- 1 rótula com 3 barras;
- Grau estático total: +5
- Grau estático interno: +6
- Grau estático externo: -1

**A04:**
- Modelo de treliça;
- Grau estático total: +1
- Grau estático interno: +1
- Grau estático externo: +0
- É possível calcular as reações de apoio para qualquer tipo de carregamento;

## Problema B

*restart*

*with(LinearAlgebra)* :

$L1 := 4.3$ :
$L2 := 5.0$ :
$L3 := 4.15$ :
$H := 3.5$ :
$q1 := 2.4$ :
$q2 := 3.8$ :
$q3 := 9.7$ :
$P := 40.0$ :
$EI := 40000.0$ :

***Determinação da Equação do Carregamento em AB***

$$A := Matrix\left(\left[[1, 0, 0], \left[1, \frac{L1}{2}, \left(\frac{L1}{2}\right)^2\right], [1, L1, L1^2]\right]\right)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2.150000000 & 4.622500000 \\ 1 & 4.3 & 18.49 \end{bmatrix} \quad (6)$$

$$B := Vector([q3, q2, q1])$$

$$\begin{bmatrix} 9.7 \\ 3.8 \\ 2.4 \end{bmatrix} \quad (7)$$

$$Vetor_qAB := MatrixInverse(A)\,B$$

$$\begin{bmatrix} 9.70000000000000 \\ -3.79069767441860 \\ 0.486749594375338 \end{bmatrix} \quad (8)$$

$$qAB := Vetor_qAB[1] + Vetor_qAB[2]\cdot x + Vetor_qAB[3]\cdot x^2$$

$$0.486749594375338\,x^2 - 3.79069767441860\,x + 9.70000000000000 \quad (9)$$

$$RES_AB := int(qAB, x = 0..L1)$$

$$19.56500000 \quad (10)$$

$$x_AB := \frac{int(qAB\cdot x, x = 0..L1)}{int(qAB, x = 0..L1)}$$

$$1.575091575 \quad (11)$$

***Cálculo da Reação de Apoio VE e do esforço interno VB***

*ΣMB_dir = 0 (antihorário)*

$$VE := solve\left(VE\cdot(L2 + L3) - \frac{q1\cdot(L2 + L3)^2}{2} + \frac{P\cdot H}{2} = 0, VE\right)$$

$$3.329726776 \quad (12)$$

*ΣMFV_BCDEF = 0 (para cima)*

$VB := solve(\mathrm{VB} - \mathrm{q1} \cdot (\mathrm{L2} + \mathrm{L3}) + \mathrm{VE} = 0, \mathrm{VB})$

$$18.63027322 \quad \textbf{(13)}$$

***Cálculo do Momento MA e Corante VA (sinal conforme a convenção para diagramas)***

$MA := -VB \cdot L1 - RES_AB \cdot x_AB$

$$-110.9268415 \quad \textbf{(14)}$$

$VA := VB + RES_AB$

$$38.19527322 \quad \textbf{(15)}$$

***Integração da Equação de Carga até a Equação da Linha Elástica***

$VAB := int(-qAB, x) + VA$

$$-0.1622498648\, x^3 + 1.895348837\, x^2 - 9.700000000\, x + 38.19527322 \quad \textbf{(16)}$$

$MAB := int(VAB, x) + MA$

$$-0.04056246620\, x^4 + 0.6317829457\, x^3 - 4.850000000\, x^2 + 38.19527322\, x - 110.9268415 \quad \textbf{(17)}$$

$\theta AB := \frac{1}{_EI} \cdot (int(MAB, x) + 0)$

$$\frac{-0.008112493240\, x^5 + 0.1579457364\, x^4 - 1.616666667\, x^3 + 19.09763661\, x^2 - 110.9268415\, x}{_EI} \quad \textbf{(18)}$$

$vAB := int(\theta AB, x) + 0$

$$\frac{1}{_EI}\left(-0.001352082207\, x^6 + 0.03158914728\, x^5 - 0.4041666668\, x^4 + 6.365878870\, x^3 - 55.46342075\, x^2\right) \quad \textbf{(19)}$$

***Momento Fletor do ponto situado no meio da barra AB***

$MAB_meioAB := subs\left(x = \frac{L1}{2}, _EI = EI, MAB\right)$

$$-45.81395200 \quad \textbf{(20)}$$

***Deflexão do ponto B***

$vAB_B := subs(x = L1, _EI = EI, vAB)$

$$-0.01549178870 \quad \textbf{(21)}$$

**Problema C** _____

$restart$
$L1 := 3.0$ :
$L2 := 4.0$ :
$L3 := 3.0$ :
$H := 4.0$ :
$q1 := 40.0$ :
$q2 := 25.0$ :
$P := 45.0$ :
$M := 50.0$ :

***Cálculo das Reações de Apoio VG, VA e HH;***

*ΣMD_dir = 0 (antihorário)*

$$VG := solve\left( VG\cdot(L1+L3) - P\cdot\left(L1+\frac{L3}{2}\right) - M - \frac{q2\cdot H^2}{2} - \frac{q1\cdot L1}{2}\cdot\frac{2}{3}\cdot L1 = 0, VG\right)$$

$$95.41666667 \quad \textbf{(22)}$$

*ΣMFV_DEFG = 0 (para cima)*

$$VD := solve\left( \mathrm{VD} - \frac{\mathrm{q1}\cdot\mathrm{L1}}{2} - \mathrm{P} + \mathrm{VG} = 0, \mathrm{VD}\right)$$

$$9.583333330 \quad \textbf{(23)}$$

*ΣMFH_DEFG = 0 (para direita)*

$$HD := solve(\mathrm{HD} - \mathrm{q2}\cdot\mathrm{H} = 0, \mathrm{HD})$$

$$100. \quad \textbf{(24)}$$

*ΣFH_ABCDH = 0 (para direita)*

$$HA := solve(\mathrm{HA} - \mathrm{HD} = 0, \mathrm{HA})$$

$$100. \quad \textbf{(25)}$$

*ΣFMD_ABCDH = 0 (horário)*

$$VA := solve\left( (VA - P)\cdot(\mathrm{L1}+\mathrm{L2}) - \frac{\mathrm{q2}\cdot(L1+L2)^2}{2} - \mathrm{HA}\cdot\mathrm{H} = 0, \mathrm{VA}\right)$$

$$189.6428571 \quad \textbf{(26)}$$

***Equação de Esforços Cortantes e Axiais da Barra AB:***

$$h1 := \sqrt{H^2 + L1^2}$$

$$5.000000000 \quad \textbf{(27)}$$

$$cos\alpha := \frac{L1}{h1}$$

$$0.6000000000 \quad \textbf{(28)}$$

$$sen\alpha := \frac{H}{h1}$$

$$0.8000000000 \quad \textbf{(29)}$$

$$VER_B := VA - P - \frac{q2\cdot L1}{2}$$

$$107.1428571$$

$$HOR_B := \mathrm{HA}$$

$$100. \quad \textbf{(31)}$$

$$FBperp := VER_B\cdot cos\alpha - HOR_B\cdot sen\alpha$$

$$-15.71428574 \quad \textbf{(32)}$$

$$FBaxial := -VER_B\cdot sen\alpha - HOR_B\cdot cos\alpha$$

$$-145.7142857 \quad \textbf{(33)}$$

$$q2perp := -q2\cdot cos\alpha^2$$

$$-9.000000000 \quad \textbf{(34)}$$

$$q2axial := q2\cdot sen\alpha\cdot cos\alpha$$

$$12.00000000 \quad \textbf{(35)}$$

$$VBC_x := FBperp + q2perp\cdot x$$

$$-9.000000000\,x - 15.71428574 \quad \textbf{(36)}$$

$$NBC_x := FBaxial + q2axial\cdot x$$

$$12.00000000\,x - 145.7142857 \quad \textbf{(37)}$$

***Cortante no Ponto F da barra EF***

$h2 := \sqrt{H^2 + L3^2}$

$$5.000000000 \quad \textbf{(38)}$$

$cos\beta := \frac{L3}{h2}$

$$0.6000000000 \quad \textbf{(39)}$$

$sen\beta := \frac{H}{h2}$

$$0.8000000000 \quad \textbf{(40)}$$

$VER_F_EF := VG - \mathrm{P}$

$$50.41666667 \quad \textbf{(41)}$$

$HOR_F_EF := -\frac{q2 \cdot H}{2}$

$$-50.00000000 \quad \textbf{(42)}$$

$\mathrm{VF_esq} := -\mathrm{HOR_F_EF} \cdot \mathrm{sen}\beta - \mathrm{VER_F_EF} \cdot \mathrm{cos}\beta$

$$9.75000000 \quad \textbf{(43)}$$

***Axial no Ponto F da barra FG***

$VER_F_FG := VG$

$$95.41666667 \quad \textbf{(44)}$$

$HOR_F_FG := -\frac{q2 \cdot H}{2}$

$$-50.00000000 \quad \textbf{(45)}$$

$\mathrm{NFdir} := +\mathrm{HOR_F_FG} \cdot \mathrm{cos}\beta - \mathrm{VER_F_FG} \cdot \mathrm{sen}\beta$

$$-106.3333333 \quad \textbf{(46)}$$

***Diagrama de Momentos Fletores Trechos GF e FE:***

$ME := VD \cdot L1 - \frac{q1 \cdot L1}{2} \cdot \frac{1}{3} \cdot L1$

$$-31.25000001 \quad \textbf{(47)}$$

$VE := +VD - \frac{q1 \cdot L1}{2}$

$$-50.41666667 \quad \textbf{(48)}$$

$HE := HD$

$$100. \quad \textbf{(49)}$$

$MEF := ME + VE \cdot cos\beta \cdot x + HE \cdot sen\beta \cdot x - \frac{q2 \cdot sen\beta^2 \cdot x^2}{2}$

$$-31.25000001 + 49.75000000\,x - 8.000000000\,x^2 \quad \textbf{(50)}$$

$MFG := MEF + M - P \cdot cos\beta \cdot \left( x - \frac{h2}{2} \right)$

$$86.24999999 + 22.75000000\,x - 8.000000000\,x^2 \quad \textbf{(51)}$$

$$MEFG := piecewise\left( \frac{h2}{2} > x, MEF, MFG \right)$$

$$\begin{cases} -31.25000001 + 49.75000000\,x - 8.000000000\,x^2 & x < 2.500000000 \\ 86.24999999 + 22.75000000\,x - 8.000000000\,x^2 & otherwise \end{cases} \tag{52}$$

***Diagrama de Esforços Cortantes Trechos GF e FE:***

$$VEF := diff(MEF, x)$$

$$49.75000000 - 16.00000000\,x \tag{53}$$

$$VFG := diff(MFG, x)$$

$$22.75000000 - 16.00000000\,x \tag{54}$$

$$VEFG := piecewise\left( \frac{h2}{2} > x, VEF, VFG \right)$$

$$\begin{cases} 49.75000000 - 16.00000000\,x & x < 2.500000000 \\ 22.75000000 - 16.00000000\,x & otherwise \end{cases} \tag{55}$$

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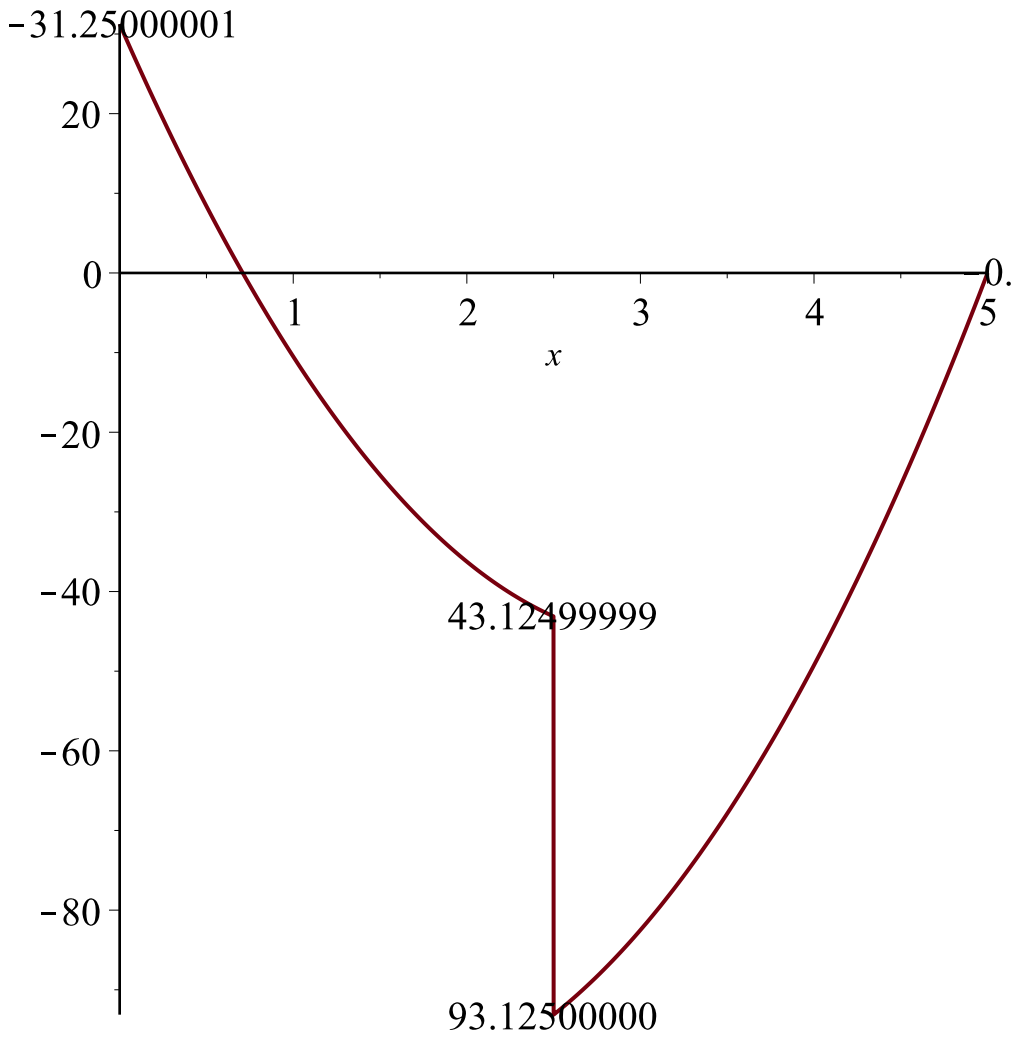
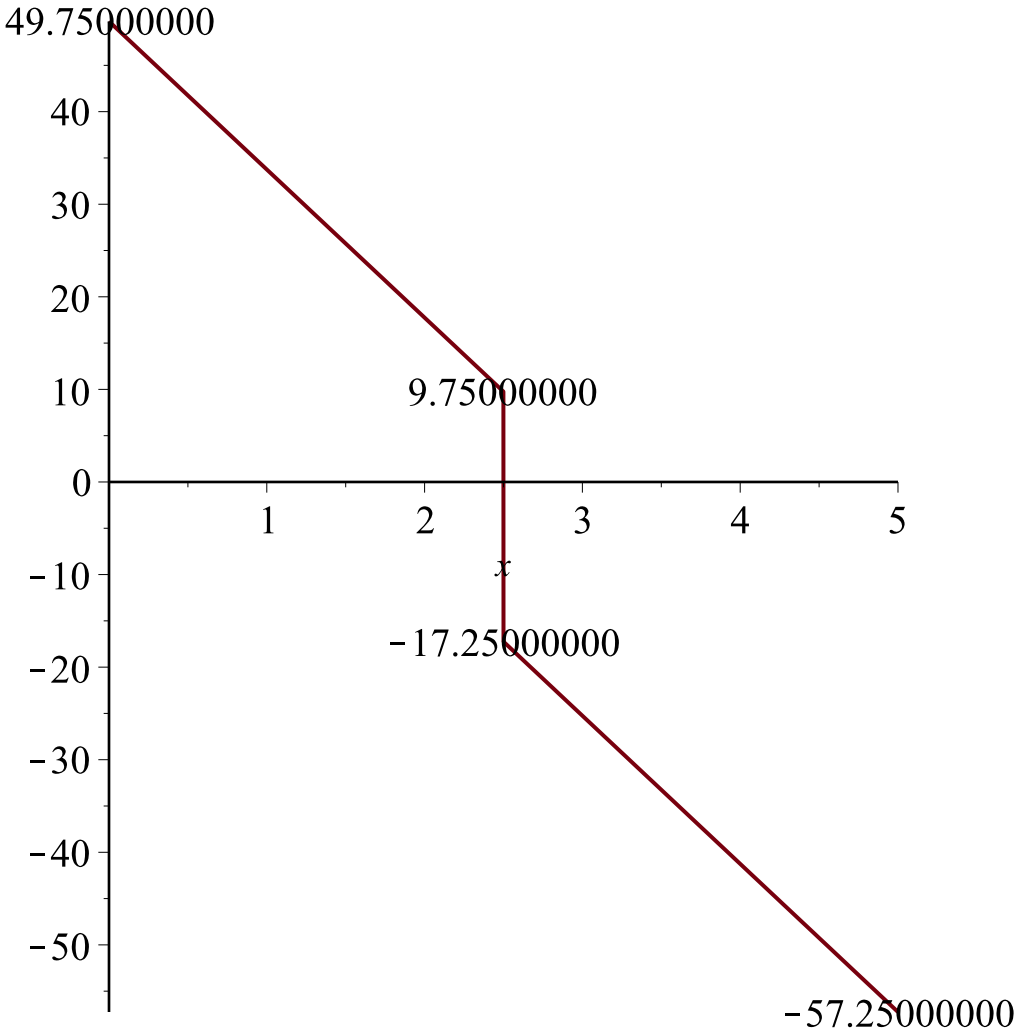
with(plots) :
mp := plot(-MEFG, x = 0..h2) :
M0 := subs(x = 0, -MEF) :
M1 := subs(x = h2/2, -MEF) :
M2 := subs(x = h2/2, -MFG) :
M3 := subs(x = h2, -MFG) :
mt0 := textplot([0, M0, 'typeset'(-M0)]) :
mt1 := textplot([h2/2, M1, 'typeset'(-M1)]) :
mt2 := textplot([h2/2, M2, 'typeset'(-M2)]) :
mt3 := textplot([h2, M3, 'typeset'(-M3)]) :

vp := plot(VEFG, x = 0..h2) :
V0 := subs(x = 0, VEF) :
V1 := subs(x = h2/2, VEF) :
V2 := subs(x = h2/2, VFG) :
V3 := subs(x = h2, VFG) :
vt0 := textplot([0, V0, 'typeset'(V0)]) :
vt1 := textplot([h2/2, V1, 'typeset'(V1)]) :
vt2 := textplot([h2/2, V2, 'typeset'(V2)]) :
vt3 := textplot([h2, V3, 'typeset'(V3)]) :
```

*display*( { *mp*, *mt0*, *mt1*, *mt2*, *mt3* } )




*display*( { *vp*, *vt0*, *vt1*, *vt2*, *vt3* } )


49.75000000
40
30
20
10
9.75000000
0
1
2
3
4
5
x
-10
-17.25000000
-20
-30
-40
-50
-57.25000000