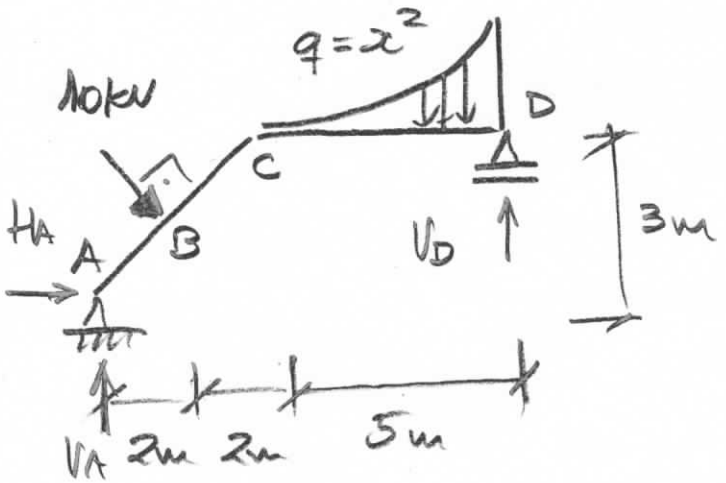
TE16 - B1 - 2000



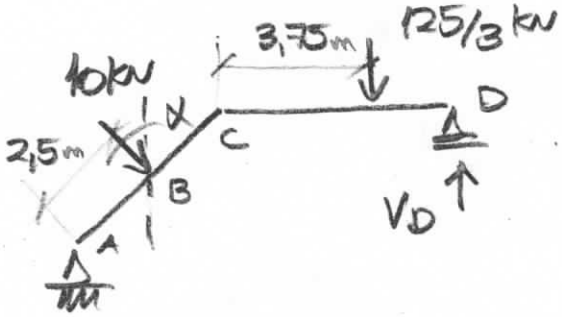
$$R_{CD} = \int_0^5 x^2 dx = \left[\frac{x^3}{3}\right]_0^5$$

$$R_{CD} = \frac{5^3}{3} = \frac{125}{3}\ kN$$

$$\bar{x}_{CD} = \frac{\int_0^5 x^2 \cdot x\, dx}{\int_0^5 x^2 dx}$$

$$\bar{x}_{CD} = \frac{\left[\frac{x^4}{4}\right]_0^5}{125/3}$$

$$\bar{x}_{CD} = \frac{5^4/4}{125/3} = \frac{625/4}{125/3} = \frac{15}{4} = 3{,}75\ m$$



$\cos\alpha = 4/5 \quad \operatorname{sen}\alpha = 3/5$

- $\sum M_A = 0$ ↺ (+)

$$V_D \times 9m - 10kN \times 2{,}5m - \frac{125}{3}kN \times 7{,}75m = 0$$

$$V_D = \frac{1}{9m}\left(25\ kN\,m + \frac{3875}{12}\ kN\,m\right) = \frac{4175}{108} = 38{,}657\ kN \uparrow$$

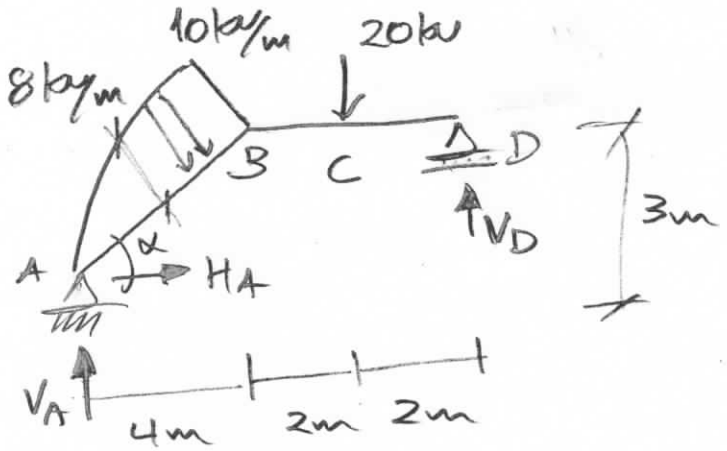
- $\sum F_V = 0$ ↑ (+) $\quad V_A + V_D - 10kN \times \frac{4}{5} - \frac{125}{3}kN = 0$

$$V_A = 8 + \frac{125}{3} - 38{,}657 = 11{,}010\ kN \uparrow$$

- $\sum F_H = 0$ → (+) $\quad H_A + 10kN \times \frac{3}{5} = 0$

$$H_A = -6kN \leftarrow$$

TE16 - B1 - 2001



$y(x) = a + bx + cx^2$

$0 = a + b\cdot 0 + c\cdot 0^2$

$8 = a + b\cdot 2{,}5 + c\cdot 2{,}5^2$

$10 = a + b\cdot 5 + c\cdot 5^2$

$a = 0$

$$\begin{cases} 2{,}5b + 2{,}5^2 c = 8 \\ 5b + 5^2 c = 10 \end{cases}$$

$b = (10 - 25c)/5 = 2 - 5c$

$2{,}5(2-5c) + 2{,}5^2 c = 8$

$5 - 12{,}5c + 6{,}25c = 8$

$c = 3/-6{,}25 = -0{,}48$

$b = +4{,}4$

$q(x) = +4{,}4x - 0{,}48x^2$

$$R_{ES} = \int_0^5 4{,}4x - 0{,}48x^2\,dx = \left[4{,}4\frac{x^2}{2} - 0{,}48\frac{x^3}{3}\right]_0^5 = \underline{35\,kN}$$

$$\bar{x} = \frac{\int_0^5 (4{,}4x - 0{,}48x^2)\,x\,dx}{\int_0^5 4{,}4x - 0{,}48x^2\,dx} = \frac{\left[4{,}4\frac{x^3}{3} - 0{,}48\frac{x^4}{4}\right]_0^5}{35\,kN} = \underline{3{,}0952\,m}$$

$\Sigma M_A = 0 \curvearrowleft \quad V_D \times 8m - 20kN \times 6m - 35kN \times 3{,}0952m$

$V_D = 1/8\,(20\times 6 + 35 \times 3{,}0952) = \underline{28{,}542\,kN}\uparrow$

$\cos\alpha = 4/5 \qquad \operatorname{sen}\alpha = 3/5$

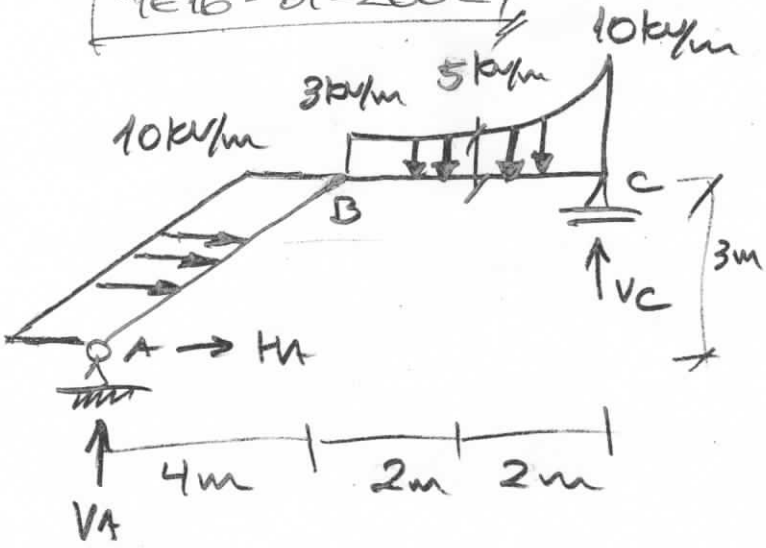
$\Sigma F_V = 0 \uparrow \quad V_A + 28{,}542kN - 20kN - 35kN \times 4/5 = 0$

$\underline{V_A = 19{,}458\,kN}\uparrow$

$\Sigma F_H = 0 \rightarrow \quad 35kN \times 3/5 + H_A = 0$

$\underline{H_A = -21\,kN} \leftarrow$

TE16 - BI - 2002



- Carga distribuída parabólica de 2º grau em BC;

→ Calcular VA, HA e VC;

$y(x) = a + bx + cx^2$

$3 = a + b\cdot 0 + c\cdot 0^2$

$5 = a + b\cdot 2 + c\cdot 2^2$

$10 = a + b\cdot 4 + c\cdot 4^2$

$$\begin{Bmatrix} 3 \\ 5 \\ 10 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 4 \\ 1 & 4 & 16 \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix}$$

$a = 3$

$b = 0{,}25$

$c = 0{,}375$

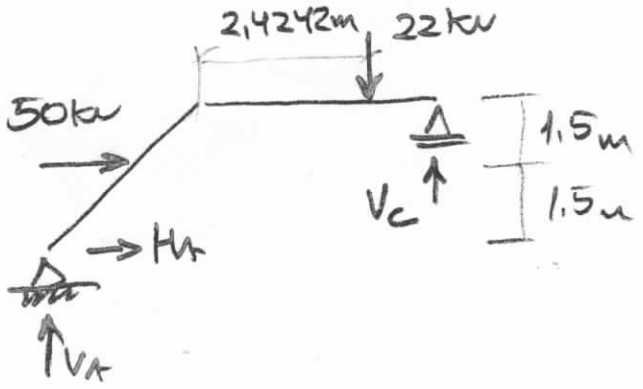
$q(x) = 3 + 0{,}25x + 0{,}375x^2$

$$Res_{BC} = \int_0^4 3 + 0{,}25x + 0{,}375x^2\,dx = \left[3x + 0{,}25\frac{x^2}{2} + 0{,}375\frac{x^3}{3}\right]_0^4$$

$Res_{BC} = 22\ kN$

$$\bar{x} = \frac{\int_0^3 (3 + 0{,}25x + 0{,}375x^2)\,x\,dx}{RES_{BD}} = 2{,}4242\ m$$

$RES_{AB} = 10\ kN/m \times 5\ m = 50\ kN$



$\Sigma M_A = 0$ (+↺)

$V_C \times 8m - 50kN \times 1{,}5m - 22kN \times 2{,}4242m = 0$

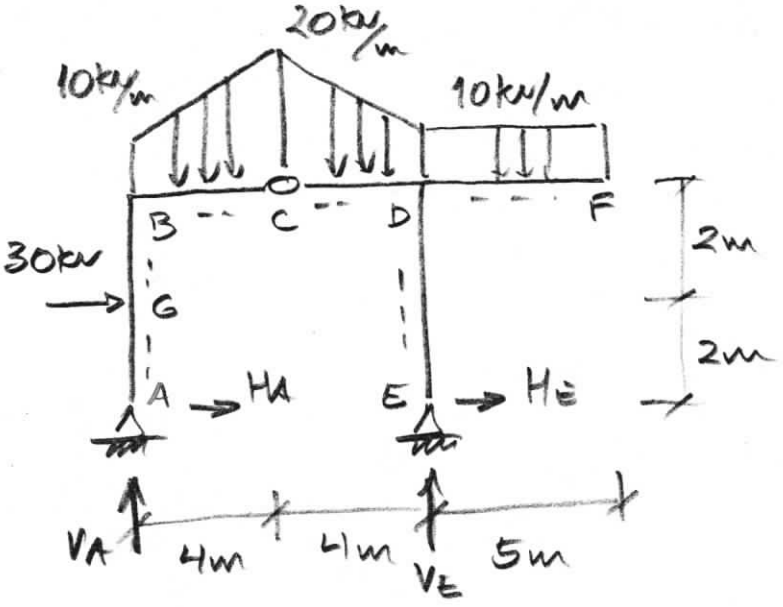
$V_C = \frac{1}{8}(50 \times 1{,}5 + 22 \times 6{,}4242)$

$V_C = 27{,}042\ kN \uparrow$

$\Sigma F_V = 0$ (+↑) $V_A + 27{,}042kN - 22kN = 0$ $\quad V_A = -5{,}042\ kN \downarrow$

$\Sigma F_H = 0$ (+→) $H_A + 50kN = 0$ $\quad H_A = -50\ kN \leftarrow$

TE16 - B1 - 3000

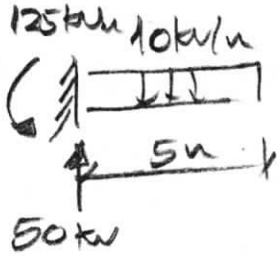


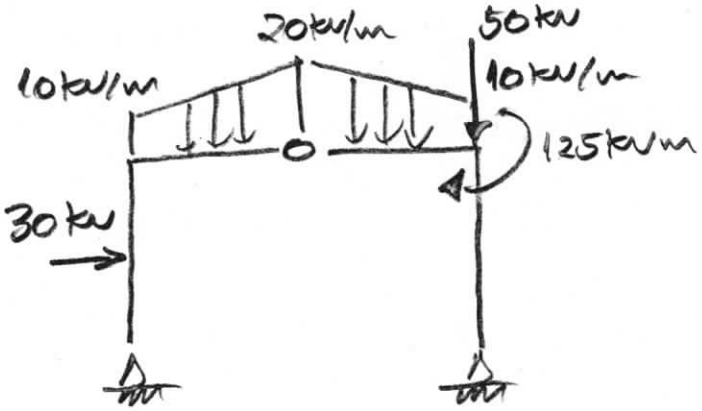
Obter a equação de Momentos Fletores do trecho CD e traçar o diagrama de Momentos Fletores p/ toda a estrutura.

- Eliminação do Balanço:

$\frac{qL^2}{2} = \frac{10 \times 5^2}{2} = 125\ kN.m$

$qL = 10 \times 5 = 50\ kN$





$\Sigma M_E = 0 \circlearrowright \oplus$

$R_{ES}_{BCD} = 2 \times (10+20)\frac{4}{2} = 120\ kN$

$V_A \times 8 - 120 \times 4 + 125 + 30 \times 2 = 0$

$V_A = \frac{1}{8}(480 - 125 - 60)$

$V_A = 36{,}875\ kN \uparrow$

$\Sigma F_V = 0\ \uparrow \oplus \quad V_A + V_E - 120 - 50 = 0$

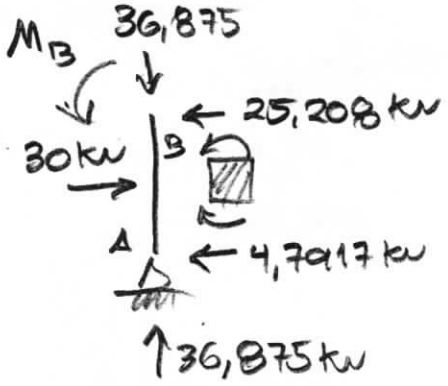
$V_E = 170 - 36{,}875 = \underline{133{,}13\ kN \uparrow}$

$\Sigma M_C^{ESQ} = 0 \circlearrowleft \oplus \quad H_A \times 4 + 30 \times 2 + 10 \times 4 \times 2 + 10 \times 4 \times \frac{1}{2} \times \frac{1}{3}4 - V_A \times 4 = 0$

$H_A = \frac{1}{4}(-60 - 80 - \frac{160}{6} + 4 \times 36{,}875) = \underline{-4{,}7917\ kN \leftarrow}$

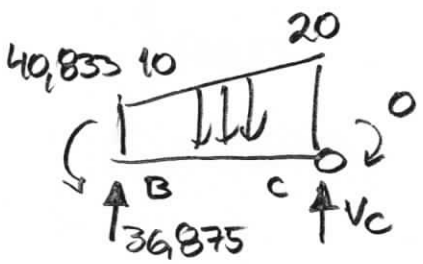
$\Sigma F_H = 0 \rightarrow \oplus \quad 30 - 4{,}7917 + H_E = 0$

$H_E = \underline{-25{,}208\ kN \leftarrow}$



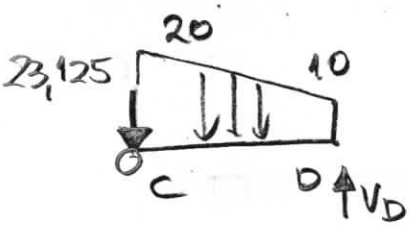
$M_B = 4,7917 kN \times 4m - 30 kN \times 2m$

$M_B = -40,833 kNm$



$V_C = 120 kN/2 - 36,875 kN = 23,125 kN$

$q_{BC}(x) = 10 + \frac{(20-10)}{4}x = 10 + \frac{5}{2}x$



$V_D = 120 kN/2 + 23,125 kN = 83,125 kN$

$q_{CD}(x) = 20 + \frac{(10-20)}{4}x = 20 - \frac{5}{2}x$

$V_{CD}(x) = -\int 20 - \frac{5}{2}x\,dx = \frac{5x^2}{4} - 20x + C_1 \qquad C_1 = V_C$

$V_{CD}(x) = \frac{5x^2}{4} - 20x - 23,125$

$M_{CD}(x) = \int \frac{5}{4}x^2 - 20x - 23,125\,dx = \frac{5x^3}{12} - \frac{20x^2}{2} - 23,125x + C_2$

$C_2 = M_C = 0$ $\qquad \boxed{M_{CD}(x) = \frac{5}{12}x^3 - 10x^2 - 23,125x}$

$V_{BC}(x) = -\int 10 + \frac{5}{2}x\,dx = -\frac{5x^2}{4} - 10x + C_1 \qquad C_1 = 36,875$

$V_{BC}(x) = -\frac{5}{4}x^2 - 10x + 36,875$

$M_{BC}(x) = \int -\frac{5}{4}x^2 - 10x + 36,875\,dx$

$M_{BC}(x) = \frac{-5}{12}x^3 - \frac{10x^2}{2} + 36,875x + C_2 \qquad C_2 = -40,833$

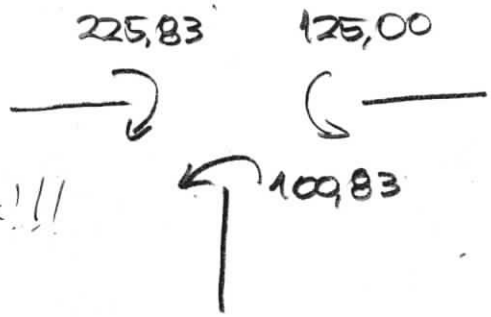
$M_{BC}(x) = \frac{-5}{12}x^3 - 5x^2 + 36,875x - 40,833$

$$M_D^{CD} = \frac{5}{12} \cdot 4^3 - 10 \cdot 4^2 - 23{,}125 \cdot 4 = -225{,}83$$



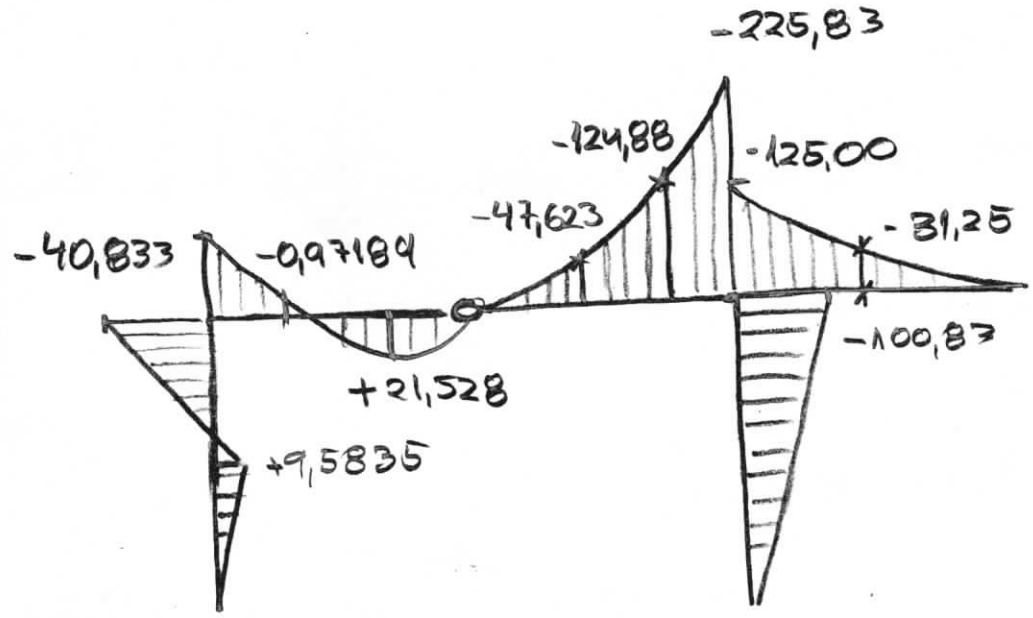
$$M_D^{DE} = 25{,}208 \times 4 = 100{,}83 \text{ kNm}$$

VERIFICAÇÃO DO PONTO D:



$$-225{,}83 + 125{,}00 + 100{,}83 = 0 \quad \text{ok!!!}$$

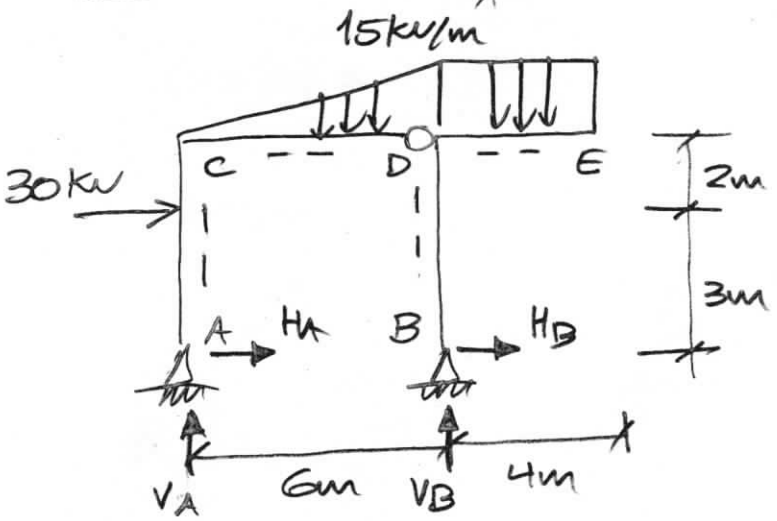
TRAÇADO DO DIAGRAMA:



$$\frac{-40{,}833}{2} + \frac{30 \times 2 \times 2}{4} = +9{,}5835 \qquad M_{BC}\left(x = \tfrac{1}{3} \cdot 4\right) =$$

$$\frac{-125}{2} + \frac{10 \times 5^2}{8} = -31{,}25$$

TEL6 - B1 - 3001



PEDE-SE:

⇒ EQUAÇÃO DE MOMENTOS FLETORES PARA O TRECHO $\bar{CD}$;

⇒ TRAÇADO DO DIAGRAMA DE CORTANTES PARA O TRECHO $\bar{AC}$;

$R_{ES}_{CD} = 15kN/m \times 6m/2 = 45kN \qquad \bar{x}_{CD} = \frac{2}{3} \times 6 = 4m$

$\Sigma M_A = 0 \curvearrowleft \oplus \quad V_B \times 6 - 30 \times 3 - 45 \times 4 - 15 \times 4 \times (6+2) = 0$

$V_B = 1/6 (90 + 180 + 480) = 125kN \uparrow$

$\Sigma F_V = 0 \uparrow \oplus \quad V_A + 125 - 45 - 15 \times 4 = 0$

$V_A = 60 + 45 - 125 = -20kN \downarrow$

$\Sigma M_D^{ESQ} = 0 \curvearrowleft \oplus \quad H_A \times 5 + 30 \times 2 + 45 \times 2 - V_A \times 6 = 0$

$H_A = 1/5 (-60 - 90 + (-20 \times 6)) = -54kN \leftarrow$

$\Sigma F_H = 0 \rightarrow \oplus \quad 30 - 54 + H_B = 0$

$H_B = 54 - 30 = 24kN \rightarrow$

$M_C = -H_A \times 5 - 30 \times 2 = +54 \times 5 - 30 \times 2 = +210kNm$

$q_{CD}(x) = \frac{15}{6} x \qquad V_{CD}(x) = -\int \frac{15}{6} x \, dx$

$V_{CD}(x) = -\frac{15}{12} x^2 + C_1 \qquad C_1 = -20kN$

$V_{CD}(x) = -\frac{15}{12} x^2 - 20$

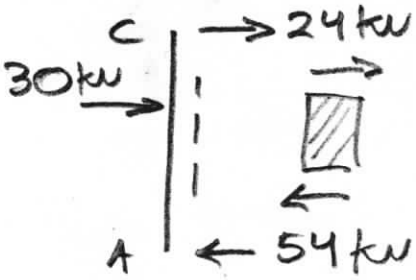
$M_{CD}(x) = \int -\frac{15}{12} x^2 - 20 \, dx = -\frac{15x^3}{36} - 20x + C_2$

$C_2 = 210kNm$

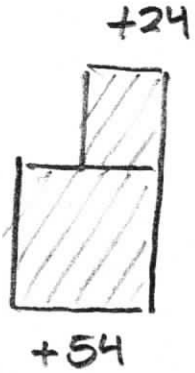
$$M_{CD}(x) = -\frac{15}{36}x^3 - 20x + 210$$

$$\boxed{M_{CD}(x) = -0{,}41667x^3 - 20x + 210}$$

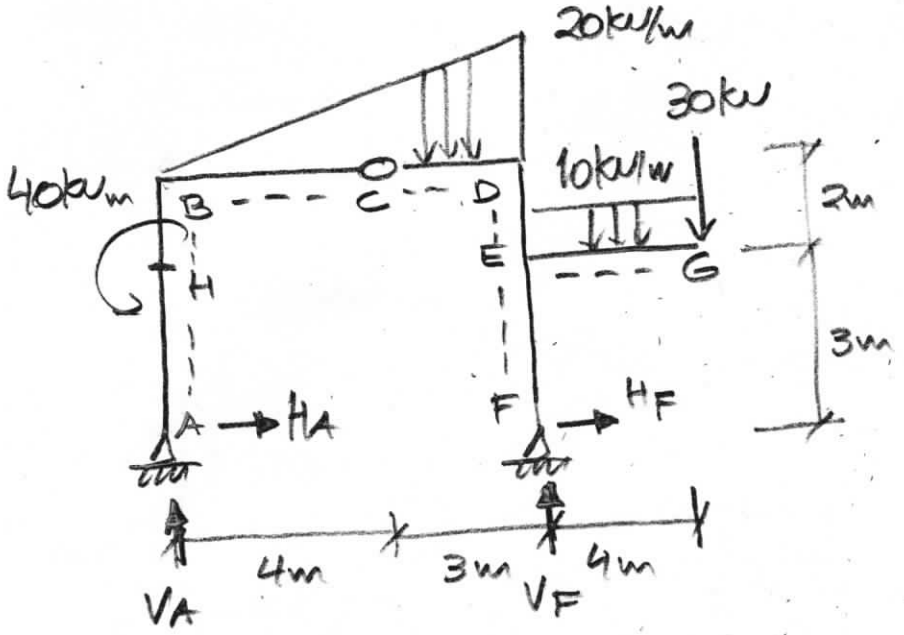
CORTANTES EM $\bar{AC}$



(V)
[kN]



TE16-B1-3002



PEDE-SE

⇒ DIAGRAMA DE MOMENTOS FLETORES DO TRECHO $\overline{DEF}$

⇒ EQUAÇÃO DE MOMENTOS FLETORES DO TRECHO $\overline{BC}$

$RES_{BCD} = 20\,kN/m \times 7m \times 1/2 = 70\,kN \quad \bar{x}_{BCD} = 2/3\,7 = 14/3$

$RES_{BC} = 20/7 \times 4\,kN/m \times 4m \times 1/2 = 22{,}86\,kN \quad \bar{x}_{BC} = 2/3\,4 = 8/3$

$\Sigma M_C^{ESQ} = 0 \circlearrowright^{+} \quad V_A \times 4 - H_A \times 5 - 40 - 22{,}86 \times 4/3 = 0$

$$4V_A - 5H_A = 70{,}48$$

$\Sigma F_H = 0 \rightarrow^{+} \quad H_A + H_F = 0$

$\Sigma F_V = 0 \uparrow^{+} \quad V_A + V_F - 30 - 10 \times 4 - 70 = 0$

$$V_A + V_F = 140$$

$\Sigma M_A = 0 \circlearrowleft^{+} \quad V_F \times 7 + 40 - 70 \times 14/3 - 40 \times 9 - 30 \times 11$

$$7V_F = 2930/3$$

$$\begin{matrix} V_A & H_A & V_F & H_F \end{matrix}$$

$$\begin{bmatrix} 4 & -5 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 7 & 0 \end{bmatrix} \begin{Bmatrix} V_A \\ H_A \\ V_F \\ H_F \end{Bmatrix} = \begin{Bmatrix} 70{,}48 \\ 0 \\ 140 \\ 2930/3 \end{Bmatrix}$$

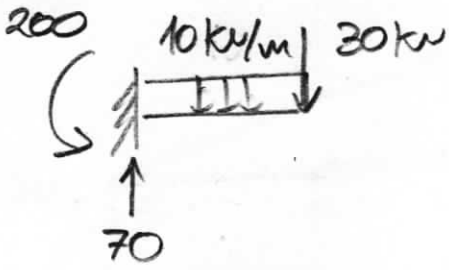
$V_A = 0{,}4762\,kN$

$H_A = -13{,}72\,kN$

$V_F = 139{,}5\,kN$

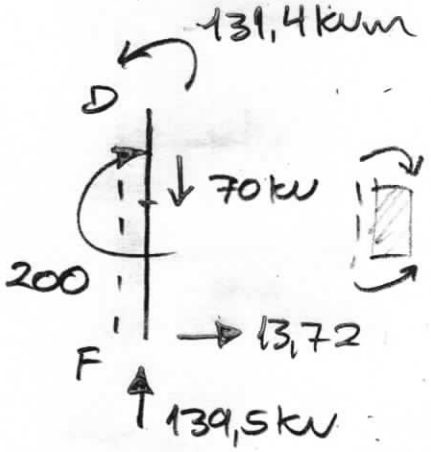
$H_F = +13{,}72\,kN$

DIAGRAMA M_{DEF}:

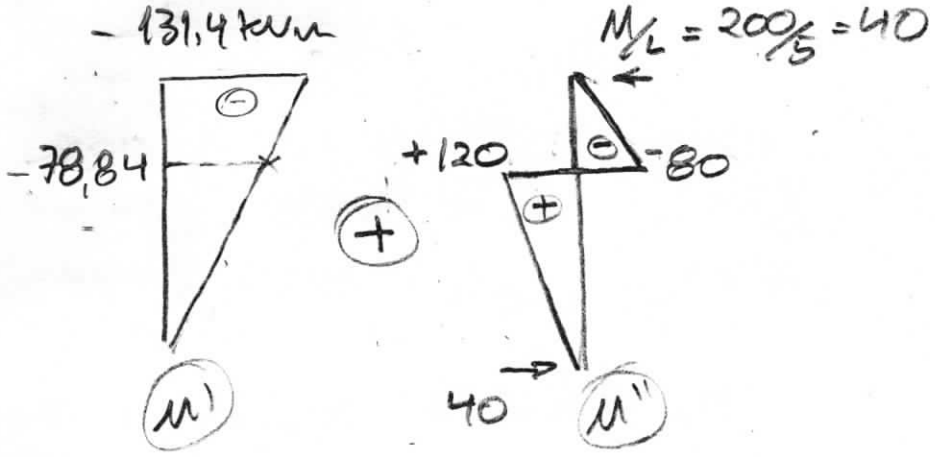


$V_E^{EG} = 10 \times 4 + 30 = 70$

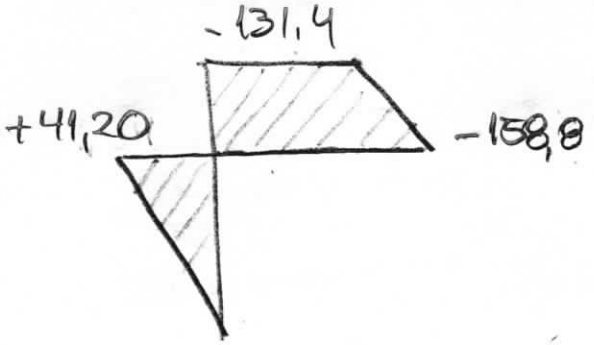
$M_E^{EG} = 30 \times 4 + 10 \times 4^2/2 = 200$



$M_D = 13{,}72 \times 5 - 200 = -131{,}4$ kNm

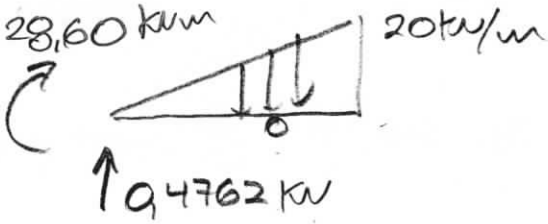


M_{FINAL} [kNm]



Eq. M_{BC}:

$M_B = -40 + 13{,}72 \times 5 = +28{,}60$



$q_{BC} = 20/7\, x$

$V_{BC}(x) = -\int 20/7\, x\, dx = -\frac{20}{14}x^2 + C_1 \qquad C_1 = 0{,}4762$

$M_{BC}(x) = \int -\frac{20}{14}x^2 + 0{,}4762\, dx = -\frac{20}{42}x^3 + 0{,}4762x + C_2$

$C_2 = +51{,}45$ $\boxed{M_{BC}(x) = -0{,}4762\,x^3 + 0{,}4762\,x + 28{,}60}$ //

TE16 - B1 - 4000

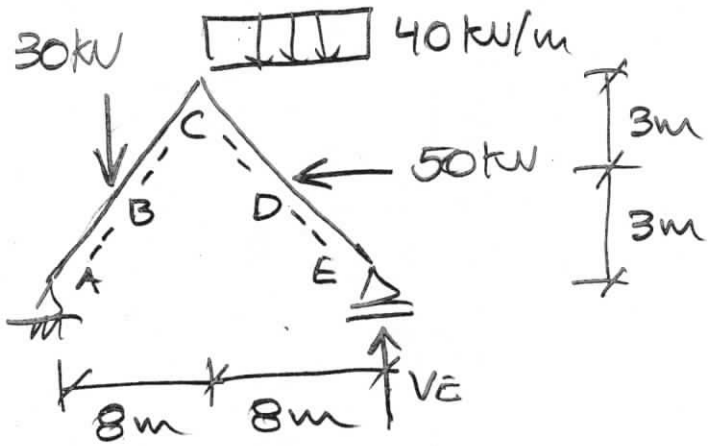
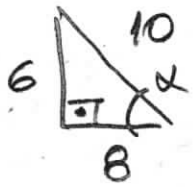


DIAGRAMA DE CORTANTES BARRA $\overline{CD}$



$\cos\alpha = 8/10$

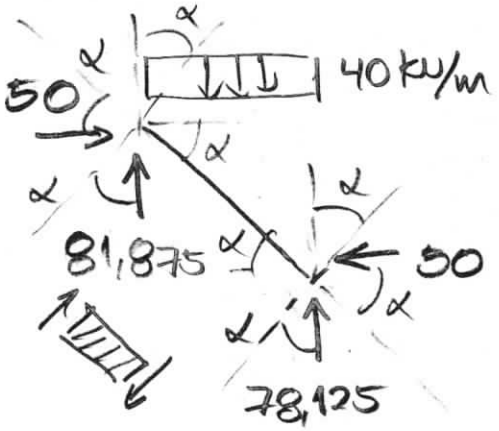
$\text{Sen}\,\alpha = 6/10$

$\sum M_A = 0 \overset{(+)}{\circlearrowleft}\quad V_E \times 16 + 50\times 3 - 30\times 4 - 40\times 8\times 12 = 0$

$V_E = 1/16\,(-150 + 120 + 320\times 12) = 238{,}125\ \text{kN}\uparrow$

$\sum F_V^{\text{con}} = 0 \overset{(+)}{\uparrow}\quad V_C - 40\times 8 + 238{,}125 = 0 \qquad V_C = +81{,}875\ \text{kN}\uparrow$

$H_C = 50\ \text{kN} \rightarrow$



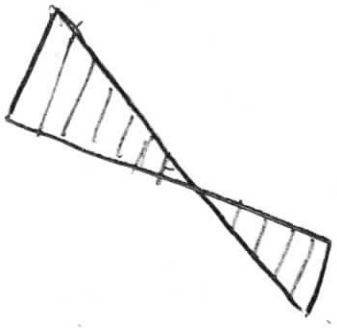
$V_D^{ESQ} = -238{,}125 + 40\times 4 = -78{,}125$

$H_D^{ESQ} = -50$

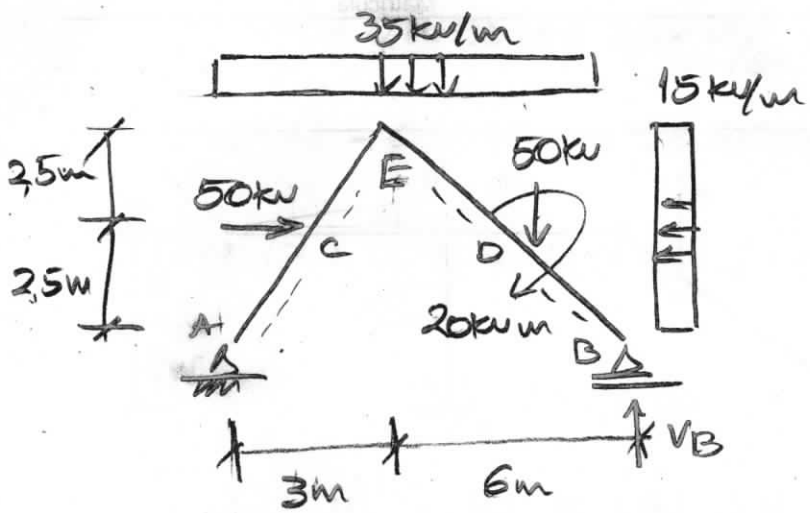
$V_{C\perp} = +50\times 6/10 + 81{,}875\times 8/10 = 95{,}5\ \text{kN}\,(+)$

$V_{D\perp}^{ESQ} = +50\times 6/10 - 78{,}125\times 8/10 = -32{,}5\ \text{kN}\,(-)$

V_{CD} [kN]



TEIG-B1-4001



• PEDE-SE:

DIAGRAMA DE ESFORÇOS CORTANTES DO TRECHO $\overline{ED}$

$$\Sigma M_A = 0 \circlearrowleft \oplus \quad V_B \times 9m + 15\frac{kN}{m} \times \frac{(5m)^2}{2} - 50kN \times 2{,}5m - 35kN/m \times \frac{(9m)^2}{2} - 20kNm - 50kN(3m+3m) = 0$$

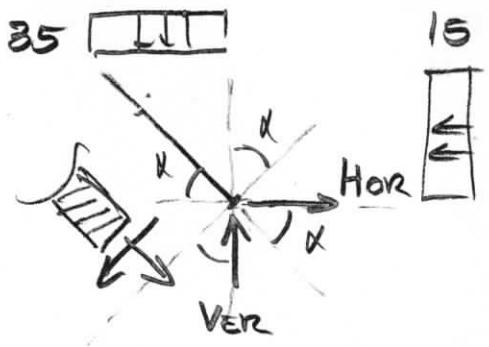
$$V_B = \frac{1}{9}\left(-\frac{15\times25}{2} + 50\times2{,}5 + \frac{35\times81}{2} + 20 + 50\times6\right) = 186{,}11\,kN\uparrow$$

$$\overrightarrow{HOR}_D^{ED} = -15\,kN/m \times 2{,}5m = -37{,}5\,kN$$

$$\uparrow VER_D^{ED} = +186{,}11\,kN - 35\,kN/m \times 3m - 50kN = +31{,}11\,kN$$



$$h = \sqrt{5^2+6^2} = \sqrt{61} \qquad \cos\alpha = 6/\sqrt{61} \qquad \operatorname{sen}\alpha = 5/\sqrt{61}$$



$$V_D^{ESQ} = -VER_D^{ED}\cos\alpha - HOR_D^{ED}\operatorname{sen}\alpha$$

$$V_D^{ESQ} = -31{,}11 \times \frac{6}{\sqrt{61}} + 37{,}5 \times \frac{5}{\sqrt{61}}$$

$$V_D^{ESQ} = 0{,}10755\,kN \quad (+)$$

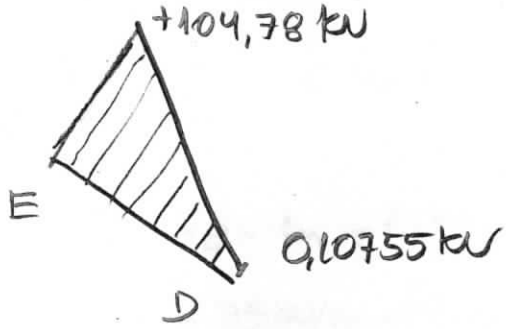
$$V_{DE}(x) = V_0^{eq} + (q_1 \cos^2\alpha + q_2 \operatorname{sen}^2\alpha)\, x$$

$$V_{DE}(x) = 0{,}10755 + (35 \times 36/61 + 15 \times 25/61)\, x$$

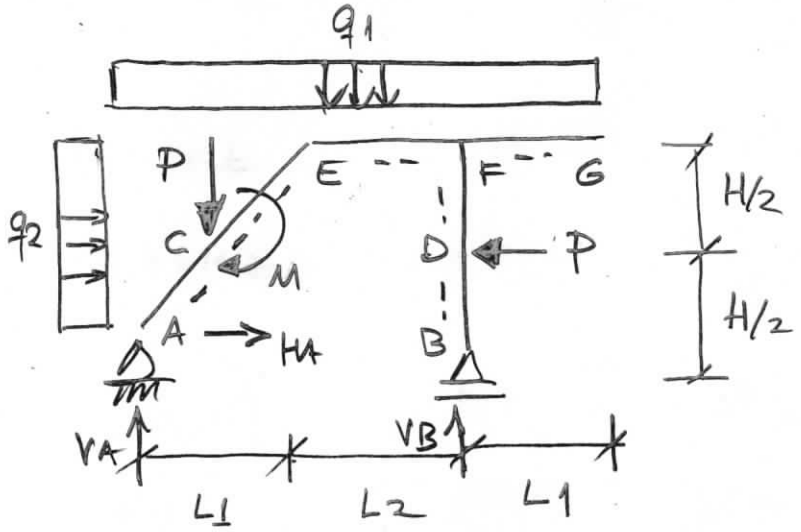
$$V_{DE}(x) = 0{,}10755 + 26{,}803\, x$$

$$V_{DE}\left(x = \frac{\sqrt{61}}{2}\right) = 0{,}10755 + 26{,}803 \times \frac{\sqrt{61}}{2} = +104{,}78\ kV$$

V_{DE}



TE16-B1-4002



$L_1 = 5m \quad L_2 = 4m$

$H = 6m \quad P = 50kN$

$M = 60kNm$

$q_1 = 20 kN/m \quad q_2 = 25 kN/m$

PEDE-SE:

- DIAGRAMA DE ESFORÇOS AXIAIS DA BARRA $\overline{CE}$

$\Sigma F_H = 0 \rightarrow \oplus \quad q_2 \times H - P + H_A = 0$

$H_A = P - q_2 \times H = 50 - 25 \times 6 = -100kN$

$\Sigma M_B = 0 \circlearrowleft \oplus \quad V_A \times (L_1 + L_2) + q_2 \frac{H^2}{2} + M - P \times H/2$

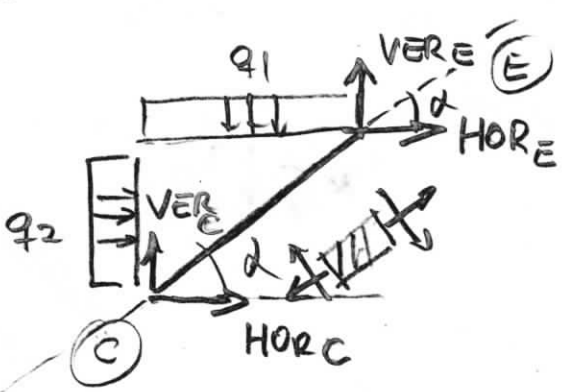
$- P(L_1/2 + L_2) - \frac{q_1 (L_1 + L_2)^2}{2} + \frac{q_1 L_1^2}{2} = 0$

$$V_A = \frac{1}{L_1 + L_2}\left[\frac{PH}{2} + P\left(\frac{L_1}{2} + L_2\right) + \frac{q_1(L_1+L_2)^2}{2} - \frac{q_1 L_1^2}{2} - M - q_2\frac{H^2}{2}\right]$$

$$V_A = \frac{1}{9}\left[\frac{50\times6}{2} + 50\left(\frac{5}{2}+4\right) + \frac{20(9)^2}{2} - \frac{20\times5^2}{2} - 60 - \frac{25\times6^2}{2}\right]$$

$V_A = 58{,}333 kN \uparrow$

* EQUILÍBRIO DE $\overline{CE}$



$N_C = -HOR_C \cos\alpha - VER_E \operatorname{sen}\alpha$

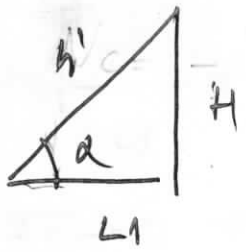
$N_E = +HOR_E \cos\alpha + VER_E \operatorname{sen}\alpha$

$\rightarrow HOR_C = HA + q2 \times H/2 = -100 + 25 \times 6/2 = -25\ kN$

$\uparrow VER_C = VA - q1 \times \frac{L1}{2} - P = 58{,}333 - 20 \times \frac{5}{2} - 50 = -41{,}667\ kN$

$\rightarrow HOR_E = -q2 \times H - HA = -25 \times 6 + 100 = -50\ kN$

$\uparrow VER_E = -VA + q1 \times L1 + P = -58{,}333 + 20 \times 5 + 50 = +91{,}667 kN$



$h' = \sqrt{L_1^2 + H^2} = \sqrt{5^2 + 6^2} = \sqrt{61}$

$\cos\alpha = L1/h' = 5/\sqrt{61}$

$\operatorname{sen}\alpha = H/h' = 6/\sqrt{61}$

$N_C = -(-25)\ 5/\sqrt{61} - (-41{,}667) \times 6/\sqrt{61} = 48{,}014\ kN$

$N_E = +(-50)\ 5/\sqrt{61} + (91{,}667) \times 6/\sqrt{61} = 38{,}411\ kN$

