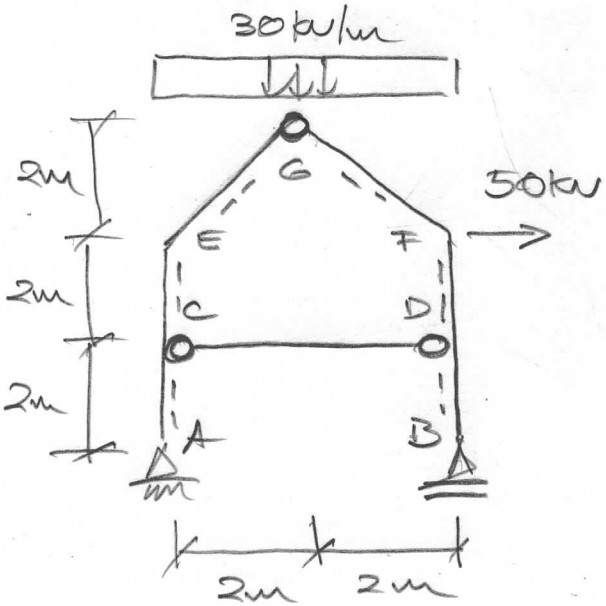
TE17 - B2 - 1000

PEDE-SE: DIAGRAMA DE CORTANTES DA BARRA EG;



$\Sigma M_A = 0 \circlearrowleft +$

$V_B \times 4m - 30kN/m \times \frac{(4m)^2}{2} - 50kN \times 4m = 0$

$V_B = \frac{1}{4}\left(30 \times \frac{16}{2} + 50 \times 4\right) = 110kN \uparrow$

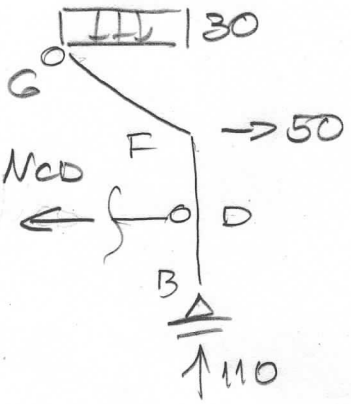
$\Sigma F_V = 0 \quad \uparrow +$

$V_A + 110kN - 30kN/m \times 4m = 0$

$V_A = -110 + 120 = 10kN \uparrow$

$\Sigma F_H = 0 \rightarrow + \quad H_A = 50kN \leftarrow$

BARRA CD:



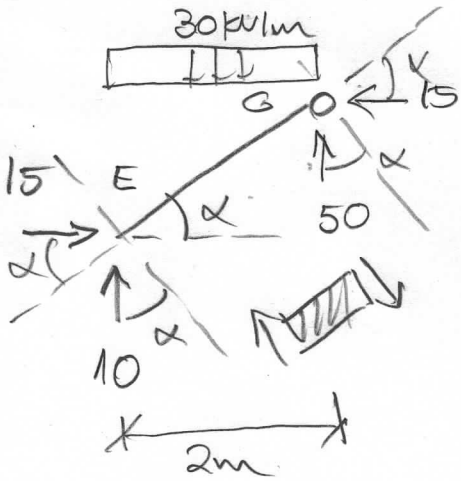
$\Sigma M_G = \circlearrowleft +$

$110kN \times 2m + 50kN \times 2m - 30\frac{kN}{m} \times \frac{(2m)^2}{2} - N_{CD} \times 4m = 0$

$N_{CD} = \frac{1}{4}\left(110 \times 2 + 50 \times 2 - 30 \times \frac{4}{2}\right) = 65kN$ (+)

$H_{OR\,E} = -50kN + 65kN = 15kN \rightarrow$

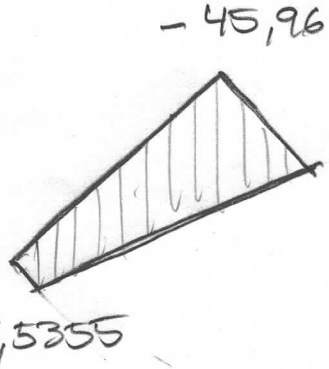
BARRA EG:



$\alpha = 45° \therefore \cos\alpha = \operatorname{sen}\alpha = \frac{\sqrt{2}}{2}$

$V_E^{EG} = 10\frac{\sqrt{2}}{2} - 15\frac{\sqrt{2}}{2} = -5\frac{\sqrt{2}}{2} = -3{,}5355kN$

$V_G^{EG} = -50\frac{\sqrt{2}}{2} - 15\frac{\sqrt{2}}{2} = -65\frac{\sqrt{2}}{2} = -45{,}962kN$

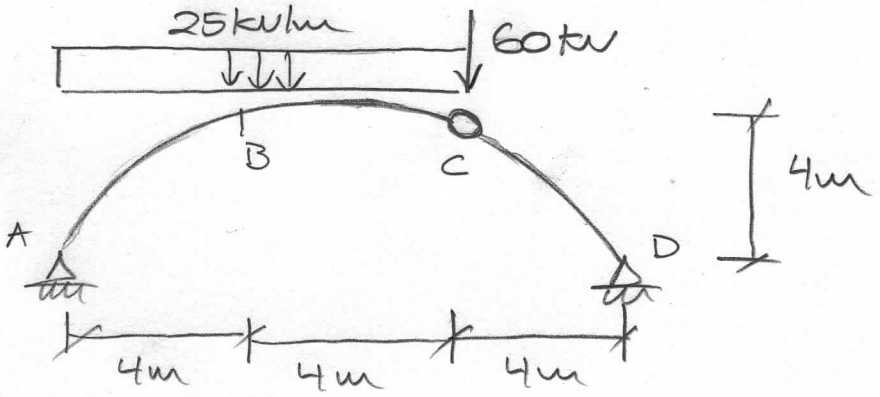


V_{EG} [kN]

TE17 - B2 - 2000

- ARCO PARABÓLICO DE 2º GRAU:

→ PEDE-SE: CORTANTE E AXIAL NO PONTO B;



$$\Sigma A = 0 \quad \circlearrowleft (+) \quad V_D \times 12m - 60kN \times 8m - 25\frac{kN}{m} \times \frac{(8m)^2}{2} = 0$$

$$V_D = \frac{1}{12}\left(60 \times 8 + 25 \times \frac{64}{2}\right) = 106{,}67\ kN \uparrow$$

$$\Sigma F_V = 0 \quad \uparrow (+) \quad V_A - 25\frac{kN}{m} \times 8m - 60kN + 106{,}67\ kN = 0$$

$$V_A = 25 \times 8 + 60 - 106{,}67 = 153{,}33\ kN \uparrow$$

$$M_{CO} = 106{,}67\ kN \times 4m = 426{,}68\ kN\,m$$

$$H = \frac{M_{CO}}{1} = \frac{426{,}68\ kN\,m}{4m} = 106{,}67\ kN\,m$$

* ASSUMINDO SISTEMA DE EIXOS COM ORIGEM EM A: A(0;0) C(8;4) D(12;0)

$$\begin{Bmatrix} 0 \\ 4 \\ 0 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0^2 \\ 1 & 8 & 8^2 \\ 1 & 12 & 12^2 \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix}$$

- FUNÇÃO DO ARCO:

$$y(x) = 1{,}5x - 0{,}125x^2$$

$$y'(x) = 1{,}5 - 0{,}25x$$

$$\alpha(x) = \operatorname{arctg}(y'(x))$$

$$\alpha(x=4) = \operatorname{arctg}(1{,}5 - 0{,}25 \times 4) = 0{,}46365$$

$\Sigma HOR_B = 160\ kN$

$\Sigma VER_B = 153{,}333\ kN - 25\ kN/m \times 4\ m = +53{,}333\ kN$

$V_B = \Sigma VER_B \cos(\alpha(4)) - \Sigma HOR \operatorname{sen}(\alpha(4))$

$V_B = +53{,}333 \times \cos(0{,}46365) - 106{,}67 \operatorname{sen}(0{,}46365)$

$\boxed{V_B = 90{,}0\ kN}$

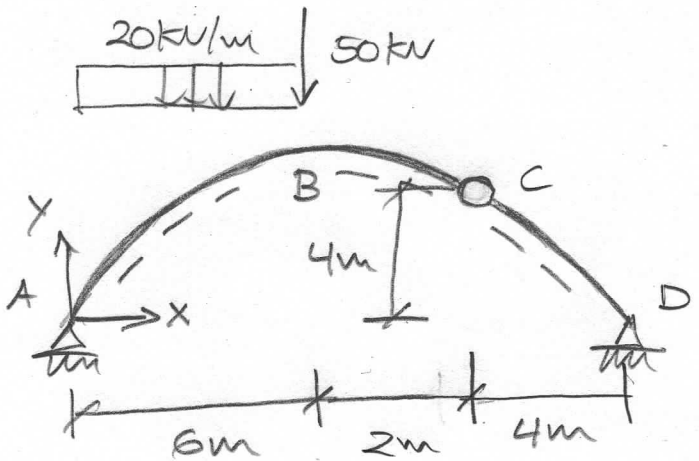
$N_B = -\Sigma VER_B \operatorname{sen}(\alpha(4)) - \Sigma HOR \cos(\alpha(4))$

$N_B = +53{,}333 \times \operatorname{sen}(0{,}46365) - 106{,}67 \cos(0{,}46365)$

$\boxed{N_B = -119{,}26\ kN}$

TE17 - B2 - 2001

→ ARCO PARABÓLICO DE 2º GRAU



→ PEDE-SE:
CORTANTE E AXIAL NO PONTO DE COORDENADA $x = 7m$

* SINAIS CONFORME CONVENÇÃO DE DIAGRAMAS;

$$\Sigma M_A = 0 \circlearrowleft (+) \quad V_D \times 12m - 20\frac{kN}{m} \times \frac{(6m)^2}{2} - 50kN \times 6m = 0$$

$$V_D = \frac{1}{12}\left(20 \times \frac{6^2}{2} + 50 \times 6\right) = \underline{55kN \uparrow}$$

$$\Sigma F_V = 0 \uparrow (+) \quad V_A - 50kN - 20kN/m \times 6m + 55kN = 0$$

$$V_A = 50 + 20 \times 6 - 55 = \underline{115kN \uparrow}$$

$$M_{CO} = 55kN \times 4m = 220kNm$$

$$H = \frac{M_{CO}}{f_{CO}} = \frac{220kNm}{4m} = \underline{55kN}$$

EQ DO ARCO: $A(0;0)$ $C(8;4)$ $D(12;0)$

$$\begin{Bmatrix} 0 \\ 4 \\ 0 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0^2 \\ 1 & 8 & 8^2 \\ 1 & 12 & 12^2 \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix}$$

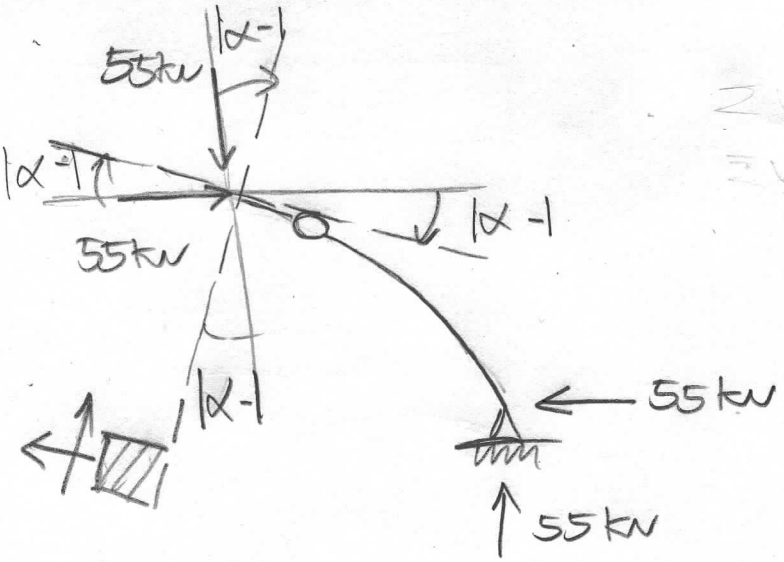
$$y(x) = 1{,}5x - 0{,}125x^2$$

$$y'(x) = 1{,}5 - 0{,}25x$$

P/ $x = 7$ $\quad \alpha(x=7) = 1{,}5 - 0{,}25 \times 7 = -0{,}25$

$\cos(\text{arctg}(-0{,}25)) = 0{,}97014$

$\text{sen}(\text{arctg}(-0{,}25)) = -0{,}24254$



$$V(x=7) = -55\cos|\alpha(x=7)| + 55\,\text{sen}\,|\alpha(x=7)|$$

$$= -55 \times 0{,}97014 + 55 \times 0{,}24254$$

$$= \underline{-40{,}018\ kN}$$

$$N(x=7) = -55\,\text{sen}\,|\alpha(x=7)| - 55\cos|\alpha(x=7)|$$

$$= -55 \times 0{,}24254 - 55 \times 0{,}97014$$

$$= \underline{-66{,}697\ kN}$$

PELAS EQUAÇÕES DE VIGA ANÁLOGA

$$V_{BC}(x=7) = 115\,kN - 20\,kN/m \times 6m - 50\,kN = -55\,kN \downarrow$$

$$H = 55\ kN$$

$$V(x=7) = V_{BC}(x=7)\cos\alpha(x=7) - H\,\text{sen}\,\alpha(x=7)$$

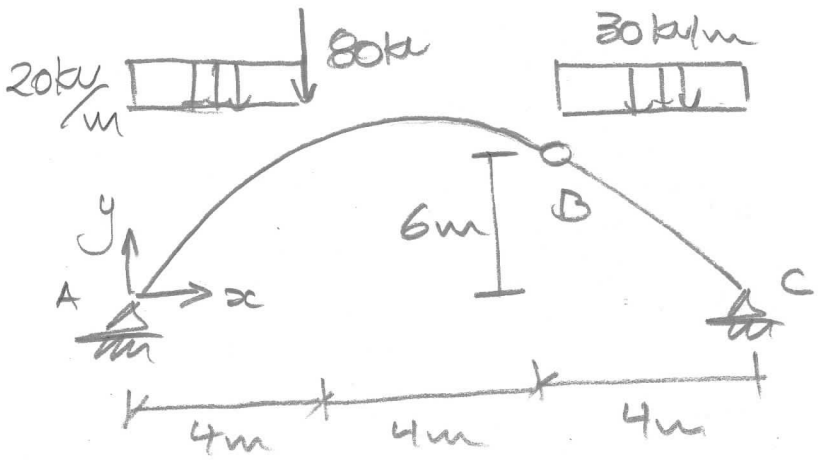
$$V(x=7) = -55 \times 0{,}97014 - 55 \times (-0{,}24254) = \underline{-40{,}018\,kN}$$

$$N(x=7) = -V_{BC}(x=7)\,\text{sen}\,\alpha(x=7) - H\cos\alpha(x=7)$$

$$N(x=7) = -(-55)(-0{,}24254) - 55 \times (0{,}97014) = \underline{-66{,}697\,kN}$$

TEI7 - B2 - 2002

ARCO PARABÓLICO DE 2º GRAU



PEDE-SE:

ESFORÇO CORTANTE E ESFORÇO AXIAL NO PONTO DE COORDENADA $x=5m$

→ Eq. da Geometria do Arco: A(0;0) B(8;6) C(12;0)

$$\begin{bmatrix} 1 & 0 & 0^2 \\ 1 & 8 & 8^2 \\ 1 & 12 & 12^2 \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{Bmatrix} 0 \\ 6 \\ 0 \end{Bmatrix}$$

$$y(x) = 2{,}25x - 0{,}1875x^2$$

$$y'(x) = 2{,}25 - 0{,}375x$$

p/ $x = 5m$ $\quad \alpha(x=5) = \text{arctg}(2{,}25 - 0{,}375 \times 5)$

$\alpha(x=5)$ arctg $(0{,}375)$

$\cos \alpha(x=5) = 0{,}93633$

$\text{Sen } \alpha(x=5) = 0{,}35112$

→ $\Sigma M_C = 0$ ↻⊕ $\quad V_A \times 12m - 30kN/m \times \frac{(4m)^2}{2} - 80kN \times 8m - 20\frac{kN}{m} \times 4m \times (8m + 2m) = 0$

$$V_A = \frac{1}{12}(30 \times 8 + 640 + 80 \times 10) = 140kN \uparrow$$

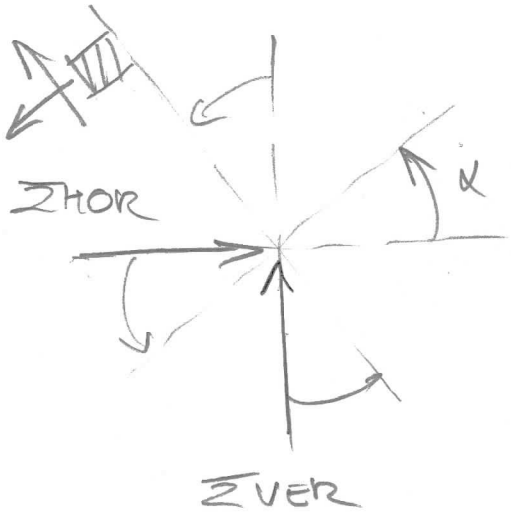
$$M_{BO} = 140kN \times 8m - 80kN \times 4m - 20\frac{kN}{m} \times 4m \times 6m$$

$$M_{BO} = 320kN/m$$

$$H_A = M_{B0}/_{1} = 320\,kNm/_{6m} = 53,333\,kN$$

$$\Sigma HOR^{ESQ}_{x=5} = 53,333\,kN \rightarrow$$

$$\Sigma VER^{ESQ}_{x=5} = 140kN - 80kN - 20\,kN/_{m} \times 4m = -20kN \downarrow$$



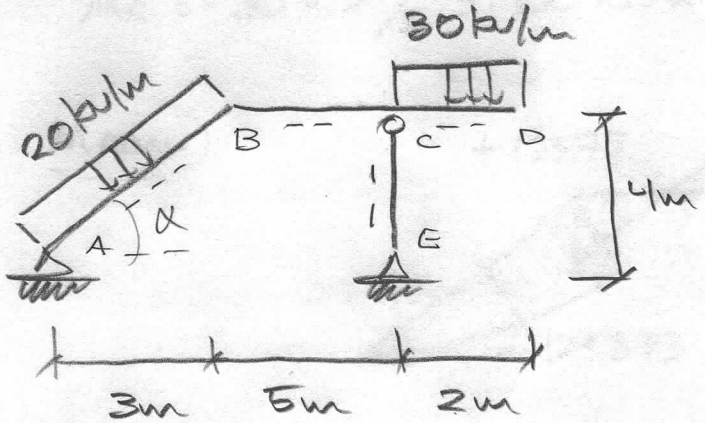
$$V(x=5) = -20 \times \cos\alpha - 53,333 \times \operatorname{sen}\alpha$$

$$= -20 \times 0,93633 - 53,333 \times 0,35112 = -37,453\,kN$$

$$N(x=5) = +20 \times \operatorname{sen}\alpha - 53,333 \times \cos\alpha$$

$$= 20 \times 0,35112 - 53,333 \times 0,93633 = -42,915\,kN$$

TEL7 - B2 - 4000



=> TODAS AS BARRAS POSSUEM SEÇÃO TRANSVERSAL RETANGULAR (15 x 40) cm

- BARRAS AB, BC E CD POSSUEM E = 50000 MPa
- BARRA CE: E = 10000 MPa

↳ CALCULAR A DEFLEXÃO DO PONTO D CONSIDERANDO EFEITOS DE ESFORÇOS AXIAIS E MOMENTOS FLETORES;

CARREGAMENTO REAL:

$\Sigma M_C^{\rightarrow E} = 0$ ↺⊕ $H_E = 0$ $\quad \cos\alpha = 3/5 \quad \operatorname{sen}\alpha = 4/5$

$\Sigma M_A = 0$ ↺⊕

$V_E \times 8m - 20\,kN/m \times (5m)^2/2 - 30\,kN/m \times 2m\,(8m + 1m) = 0$

$V_E = 1/8\,(10 \times 25 + 30 \times 2 \times 9) = \underline{98{,}75\,kN} \uparrow$

$q_{AB\,VER} = 20\,kN/m \times 5m \times 3/5 \times 1/3m = 20\,kN/m$

$q_{AB\,HOR.} = 20\,kN/m \times 5m \times 4/5 \times 1/4m = 20\,kN/m$

$\Sigma F_{HOR} = 0$ →⊕ $\quad H_A = -20\,kN/m \times 4m = \underline{-80\,kN} \leftarrow$

$\Sigma F_{VER} = 0$ ↑⊕

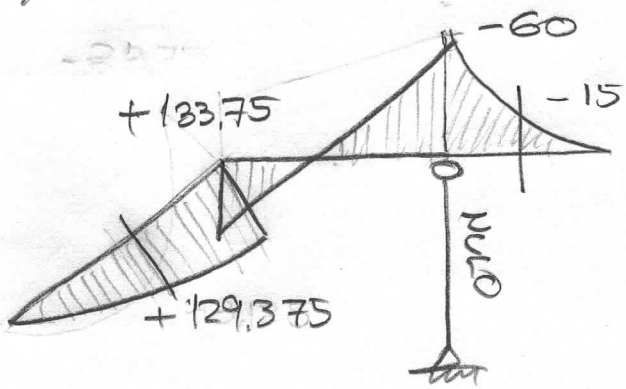
$V_A - 20\,kN/m \times 3m - 30\,kN/m \times 2m + 98{,}75\,kN = 0$

$V_A = 60 + 60 - 98{,}75 = \underline{21{,}25\,kN} \uparrow$

$M_{IB} = 80,0 \times 4 + 21,25 \times 3 - 20 \times 5^2/2 = +133,75 \text{ kNm}$

$M_C = -30 \times 2^2/2 = -60 \text{ kNm}$

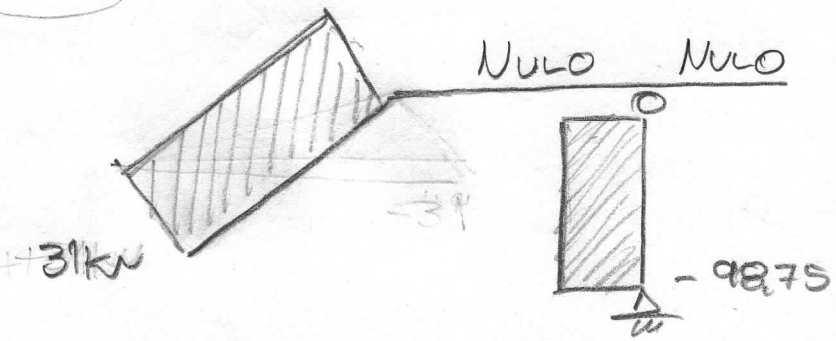
M_{REAL}



$\frac{-60+0}{2} + 30 \times 2^2/8 = -15 \text{ kNm}$

$\frac{+133,75+0}{2} + 20 \times 5^2/8 = 129,375 \text{ kNm}$

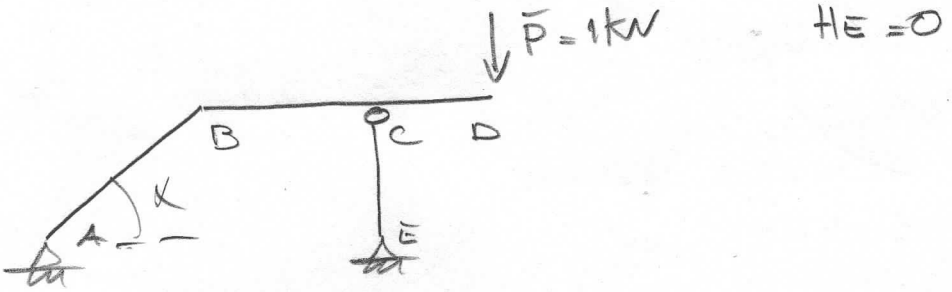
N_{REAL}



$N_A = +80 \times \cos\alpha - 21,25 \times \text{sen}\alpha =$

$= 80 \times 3/5 - 21,25 \times 4/5 = 31 \text{ kN}$

CARREGAMENTO VIRTUAL

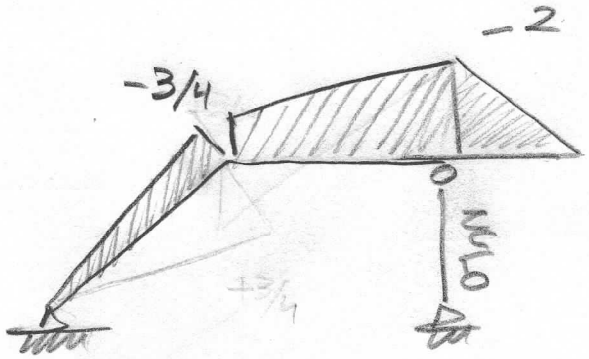


$\Sigma M_A = 0 \circlearrowleft \quad V_E \times 8m - 1kN \times 10m = 0$

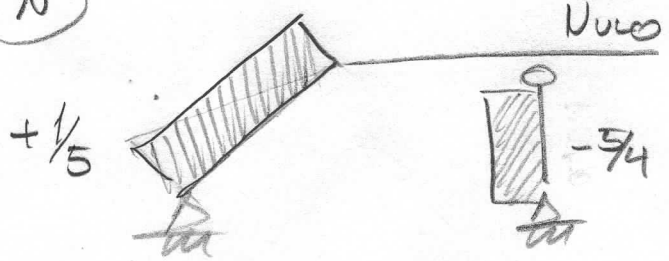
$V_E = 5/4\ kN$

$\Sigma F_V = 0 \uparrow \quad V_A - 1 + 5/4 = 0 \qquad V_A = -1/4\ kN$

$\bar{M}$



$\bar{N}$



$$\delta_{VD\,MOM.} = \frac{1}{EI_{AD}}\left[\frac{1}{6}\times 5\times(-0{,}75)\left(2\times 129{,}375 + 133{,}75\right)\right]$$

$$\frac{1}{EI_{AD}}\times\frac{5}{6}\left[133{,}75\left(2\times(-0{,}75)-2\right)-60\left(-0{,}75+2(-2)\right)\right] +$$

$$\frac{1}{EI_{AD}}\times\left[\frac{1}{6}\times 2\times -2\left(2\times(-15)-60\right)\right] = \frac{-245{,}31-152{,}60+60}{EI_{AD}}$$

$$EI_{AD} = 50\times10^{6}\,kN/m^2\times 0{,}15\,m\times\frac{(0{,}4\,m)^3}{12} = 40000\,kNm^2$$

$$\delta_{VD\,MOM} = \frac{-245{,}31-152{,}6+60}{40000} = -0{,}0084478\,m \uparrow$$

$$\delta_{VD\,AXIAL} = \left[5\times 31\times\frac{1}{5} + 4\times(-98{,}75)\times(-1{,}25)\right]\frac{1}{EA_{CE}}$$

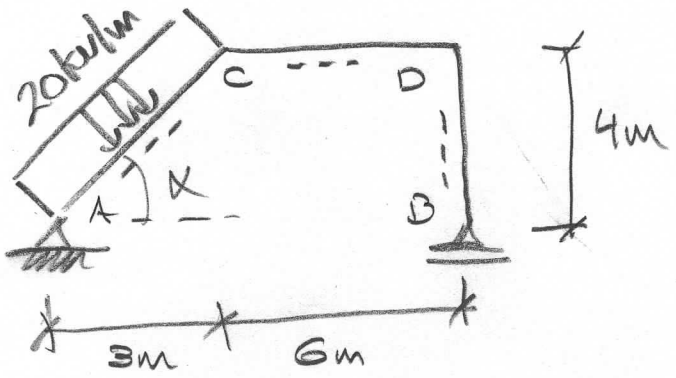
$$EA_{CE} = 10\times10^{6}\,kN/m^2\times 0{,}15\times 0{,}4\,m^2 = 600000\,kN$$

$$\delta_{VD\,AXIAL} = \frac{31+493{,}75}{600000} = +0{,}00087458\,m \downarrow$$

$$\delta_{TOTAL} = -8{,}4478\times10^{-3}\,m + 8{,}7458\times10^{-4}\,m$$

$$= -7{,}573\times10^{-3}\,m = 7{,}573\,mm \uparrow$$

TE17-B2-4001



PARA TODAS AS BARRAS:

$E = 30000\ MPa$

base = 20 cm

altura = 40 cm

PEDE-SE:

DESLOCAMENTO HORIZONTAL DO PONTO D EM "mm" E COM ⊕ P/ DIREITA;

* OBS: CONSIDERAR MOM. FLETORES E AXIAIS NOS CÁLCULOS;

Carregamento Real: $\cos\alpha = 3/5$ $\operatorname{sen}\alpha = 4/5$

$\sum M_A = 0$ ↺ ⊕ $\quad V_B \times 9m - 20kN/m \times (5m)^2/2 = 0$

$V_B = 1/9 \times 20 \times 25/2 = 27{,}778\ kN \uparrow$

- Decompondo a dist. q em horiz. (q_H) e vert. (q_V)

$q_H = 20\,kN/m \times 5m \times \cos\alpha \times \frac{1}{3m} = 20 \times 5 \times 3/5 \times 1/3 = 20\,kN/m$

$q_V = 20\,kN/m \times 5m \times \operatorname{sen}\alpha \times \frac{1}{4m} = 20 \times 5 \times 4/5 \times 1/4 = 20\,kN/m$

LOGO:

$\sum V = 0 \uparrow \quad V_A - 20kN/m \times 3m + 27{,}778\,kN = 0$

$V_A = 60 - 27{,}778 = 32{,}222\ kN \uparrow$

$\sum H = 0 \rightarrow \quad H_A + 20kN/m \times 4m = 0$

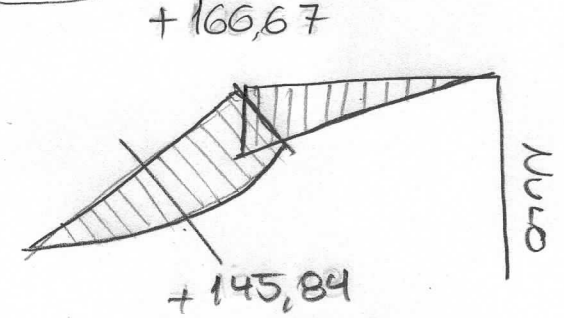
$H_A = -80\,kN \leftarrow$

$M_D = 0$

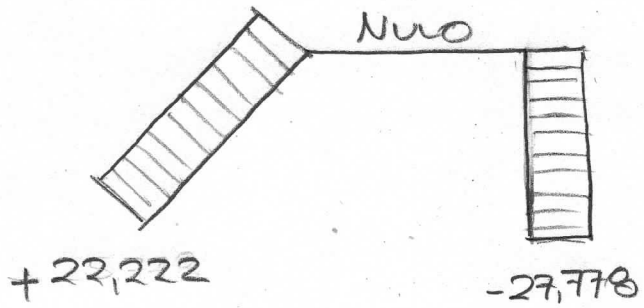
$M_C = 80\,kN \times 4m + 37{,}222\,kN \times 3m - 20\,kN/m \times (5m)^2/2$

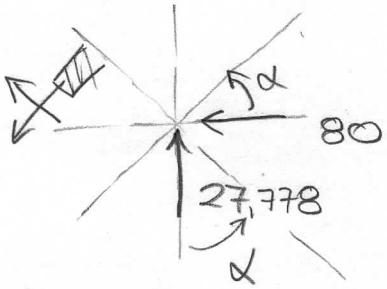
$= 166{,}67\,kN\,m$

M_{REAL}



N_{REAL}



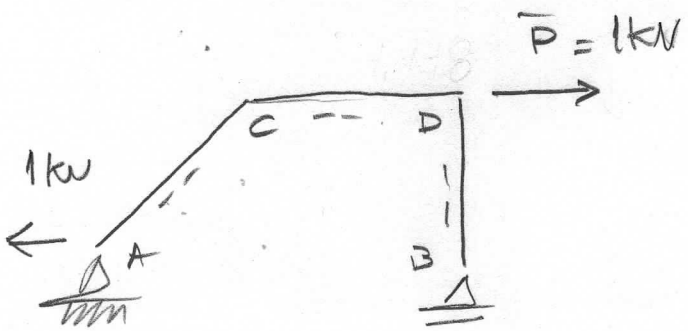


$N_A = 80 \times \cos\alpha - 37{,}222 \times \operatorname{sen}\alpha$

$N_A = 80 \times 3/5 - 37{,}222 \times 4/5 = 22{,}222\,kN$

$\frac{166{,}67 + 0}{2} + 20 \times 5^2/8 = 145{,}84\,kN\,m$

Carregamento Virtual:



$\rightarrow \Sigma F_H = 0 \quad \bar{H}_A = -1\,kN\ (\leftarrow)$

$\Sigma M_A = 0 \circlearrowleft \oplus$

$V_B \times 9m - 1\,kN \times 4m = 0$

$V_B = 4\,kN\,m/9m = 4/9\,kN \uparrow$

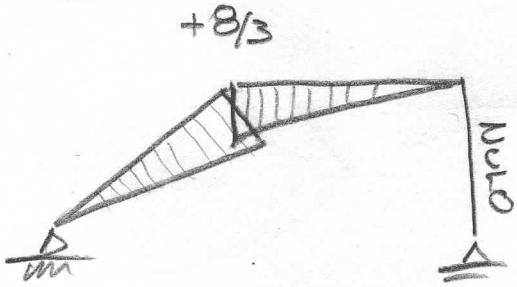
$\Sigma F_V = 0 \uparrow \oplus \quad V_A + V_B = 0$

$V_A = -4/9\,kN \downarrow$

$M_{DB}(x) = 0$

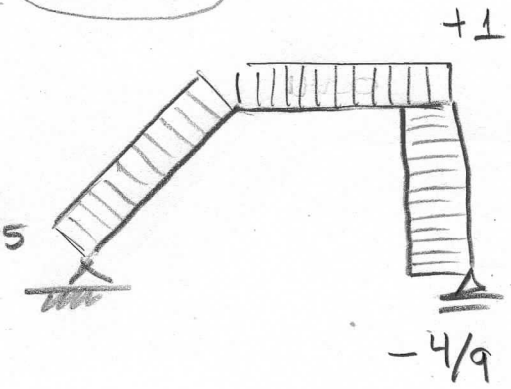
$M_C = -4/9\,kN \times 3m + 1kN \times 4m = 8/3\,kNm$

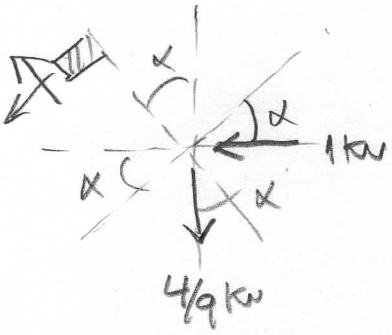
$\bar{M}$ VIRTUAL



$+ 11/45$

$\bar{N}$ VIRTUAL





$\bar{N}_A = 1kN \times \cos\alpha + 4/9\,kN \sin\alpha$

$\bar{N}_A = 1 \times 3/5 - 4/9 \times 4/5 = 0,24444\,kN +$

$$\delta_{HD\ MOM} = \frac{1}{EI}\left[\frac{1}{6} 5 \times \frac{8}{3}(2 \times 145,84 + 166,67) + \frac{1}{3} 6 \times \frac{8}{3} \times 166,67\right]$$

$$EI = 30 \times 10^6\,kN/m^2 \times 0,2m \times \frac{(0,4m)^3}{12} = 32000$$

$$\delta_{HD\ MOM} = \frac{1018,6 + 888,91}{32000} = 0,059609m = 59,609mm$$

$$\delta_{HD\ AXIAL} = \frac{1}{EA}\left[5 \times 22,222 \times \frac{11}{45} + 4 \times (-27,778)(-4/9)\right]$$

$$EA = 30 \times 10^6\,kN/m^2 \times 0,2m \times 0,4m = 2400000$$

$$\delta_{HD\ AXIAL} = \frac{27,160 + 49,383}{24000000} = 3,1893 \times 10^{-6}m = 0,003189mm$$

$\delta_{HD} = 59,612mm \rightarrow$ //