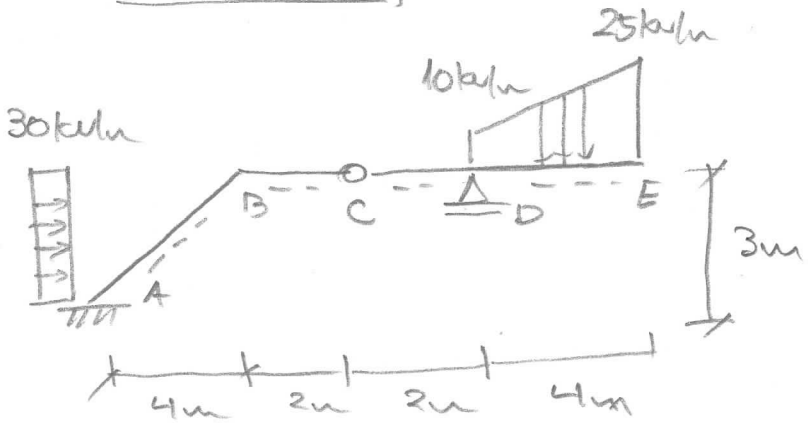
BZTEOGAMA



PEDE-SE:

Q01) MOMENTO FLETOR NO MEIO DA BARRA AB;

Q02) CORTANTE NO MEIO DA BARRA AB;

Q03) AXIAL NO MEIO DA BARRA AB;

$q_{DE}(x) = 10 + \frac{25-10}{4}x = 10 + 3{,}75x$ $\quad V_{DE}(x) = \int -q_{DE}(x)\,dx$

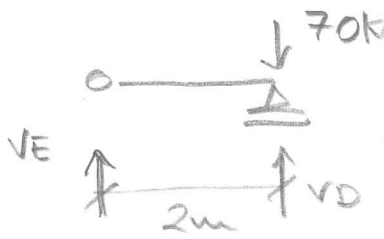
$V_{DE}(x) = -\frac{3{,}75x^2}{2} - 10x + C_1$ $\quad$ p/ $x = 4$ $\quad V = 0$ Logo:

$\frac{3{,}75}{2} \times 4^2 + 10 \times 4 + C_1 = 0 \rightsquigarrow \underline{C_1 = +70}$

$M_{DE}(x) = \int V_{DE}(x)\,dx = -3{,}75\,x^3/3 - 10x^2/2 + 70x + C_2$

P/ $x = 4$ $\quad M = 0$ Logo:

$C_2 = +3{,}75 \times 4^3/6 + 10/2 \times 4^2 - 70 \times 4 = \underline{-160}$



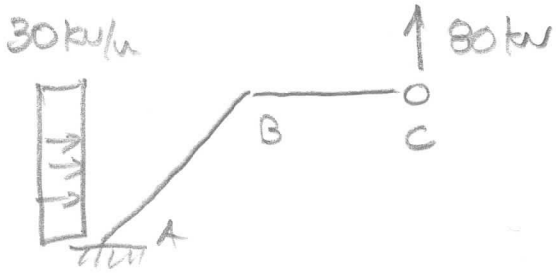
$\Sigma M_E = 0$ ↺ ⊕

$V_D \times 2 - 70 \times 2 - 160 = 0$

$V_D = 1/2\,(140 + 160) = 150\,kN \uparrow$

$\Sigma F_V = 0$ ↑⊕ TRECHO CDE

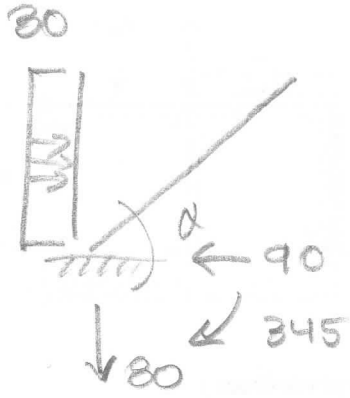
$V_E + 150kN - 70kN = 0$ $\quad V_E = -150 + 70 = -80kN \downarrow$



$\Sigma F_V^{ABC} = 0$ $\quad V_A = -80kN \downarrow$

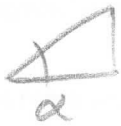
$\Sigma F_h^{ABC} = 0$ $\quad H_A = 30 \times 3 = -90kN \leftarrow$

$M_A = -80 \times 6 + 30 \times 3^2/2 = \underline{-345\,kNm}$ ↻



$$N_{AB\,MEIO} = 345 - 80\times\tfrac{4}{2} + 90\times\tfrac{3}{2} - 30\times\left(\tfrac{3}{2}\right)^2\times\tfrac{1}{2} = 286{,}25\ kN$$

Q01



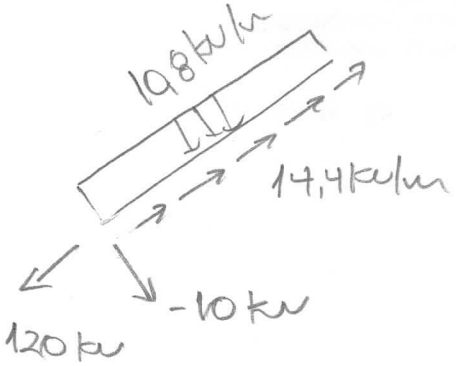
$\cos\alpha = 4/5$

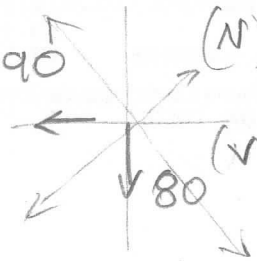
$\operatorname{sen}\alpha = 3/5$

DECOMPOSIÇÃO EM PERP. E PARALELO:

$$q_{PERP} = 30\times\operatorname{sen}^2\alpha = 30\times\left(\tfrac{3}{5}\right)^2 = 10{,}8\ kN/m$$

$$q_{PAR} = 30\times\operatorname{sen}\alpha\times\cos\alpha = 30\times\tfrac{12}{25} = 14{,}4\ kN/m$$





$$\begin{Bmatrix} -120 \\ -10 \end{Bmatrix} = \begin{bmatrix} 4/5 & 3/5 \\ -3/5 & 4/5 \end{bmatrix} \begin{Bmatrix} -90 \\ -80 \end{Bmatrix}$$

$$N_{AB\,MEIO} = 120\ kN - 14{,}4\times 2{,}5 = +84\ kN \quad \textbf{Q03}$$

$$V_{AB\,MEIO} = -10 - 10{,}8\times 2{,}5 = -37\ kN \quad \textbf{Q02}$$

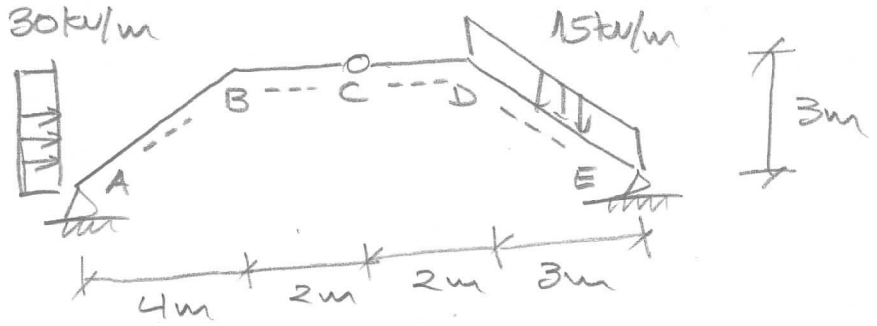
B2 TEQ6ANC

Q01: M NO MEIO DE $\overline{AB}$;

Q02: V NO MEIO DE $\overline{AB}$;

Q3: N NO MEIO DE $\overline{DE}$;

$L_{DE} = \sqrt{18}$



REAÇÕES DE APOIO: $\Sigma M_A = 0$ ↺ ⊕

$V_E \times 11 - 30 \times 3^2/2 - 15 \times \sqrt{18} \times 9{,}5 = 0$

$V_E = 1/11 \left(15 \times 9 + 15 \times \sqrt{18} \times 9{,}5\right) = \underline{67{,}234\ kN} \uparrow$

$\Sigma F_V = 0$ ↑ ⊕ $V_A - 15 \times \sqrt{18} + 67{,}234\ kN = 0$

$V_A = 15 \times \sqrt{18} - 67{,}234 = \underline{-3{,}5944\ kN} \downarrow$

$\Sigma M_C^{\rightarrow E} = 0$ ↺ ⊕ $H_E \times 3 - 15\sqrt{18} \times 3{,}5 + 67{,}234 \times 5 = 0$

$H_E = 1/3\left(15 \times \sqrt{18} \times 3{,}5 - 67{,}234 \times 5\right) = \underline{-37{,}811\ kN} \leftarrow$

$\Sigma F_H = 0$ → ⊕ $H_A + 30 \times 3 - 37{,}811 = 0$

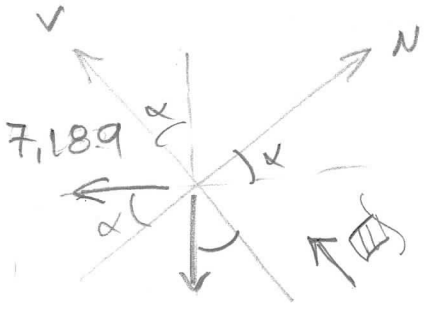
$H_A = -30 \times 3 + 37{,}811 = \underline{-52{,}189\ kN} \leftarrow$

$M_{AB\ MEIO} = -3{,}5944 \times 2 - 30 \times 1{,}5^2/2 + 52{,}189 \times 1{,}5$

$= \underline{+37{,}345\ kN}$ Q01

$\bar{V}_{AB\ MEIO} = -3{,}5944$ $\bar{H}_{AB\ MEIO} = 30 \times 1{,}5 - 52{,}189$

$= -7{,}189\ kN$



$\cos\alpha = 4/5$ $\operatorname{sen}\alpha = 3/5$

$V_{AB\ MEIO} = 7{,}189\ \operatorname{sen}\alpha - 3{,}5944 \cos\alpha$

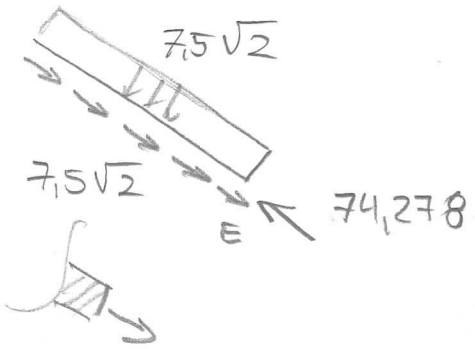
$= 7{,}189 \times 3/5 - 3{,}5944 \times 4/5$

$= \underline{+1{,}4379\ kN}$ Q02

BARRA DE:

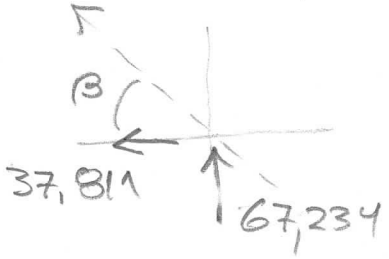
$q\perp = 15 \times \sqrt{2}/2 = 7{,}5\sqrt{2}$

$q// = 15 \times \sqrt{2}/2 = 7{,}5\sqrt{2}$



$L = \sqrt{3^2+3^2} = \sqrt{18}$

PONTO E

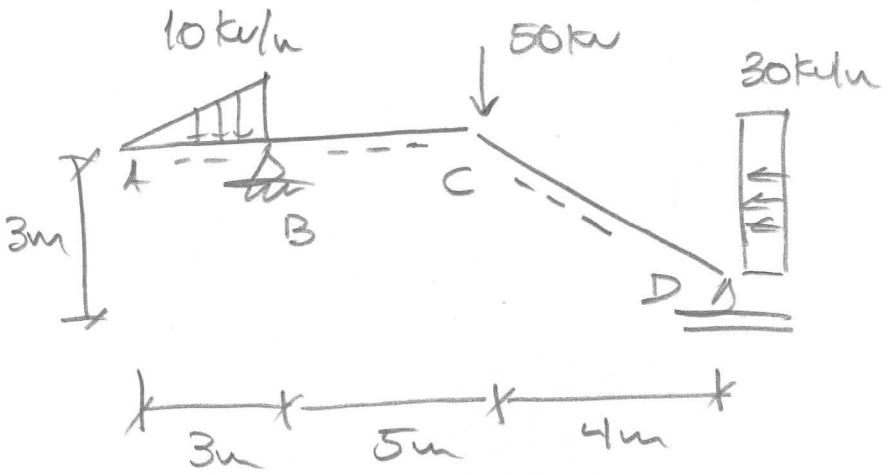


$N_E = +37{,}811 \times \sqrt{2}/2 + 67{,}234 \times \sqrt{2}/2$

$= 74{,}278 kN$

$N_{DE\ MEIO} = -74{,}278 + 7{,}5\sqrt{2} \times \sqrt{18}/2 = \underline{-51{,}778\ kN}$ Q03

B2 TEØ6 AND VALOR 1,0



Q01) CORTANTE NO MEIO DA BARRA CD;

Q02) AXIAL NO MEIO DA BARRA CD;

Q03) MOMENTO NO MEIO DA BARRA BC

$\Sigma M_B = 0 \;\curvearrowleft\oplus\; V_D \times 9 - 50\times 5 - 30\times 3\times 1{,}5 + \frac{10\times 3}{2}\times 1 = 0$

$V_D = \frac{1}{9}(50\times 5 + 90\times 1{,}5 - 15\times 1) = \underline{41{,}111\ kN}\ \uparrow$

$\Sigma F_V = 0 \uparrow\oplus\; V_B - \frac{10\times 3}{2} - 50 = 0$

$V_B = 15 + 50 = \underline{65\ kN}\ \uparrow$

$\Sigma F_H = 0 \rightarrow\oplus \qquad H_B = 30\times 3 = \underline{90\ kN}\ \rightarrow$

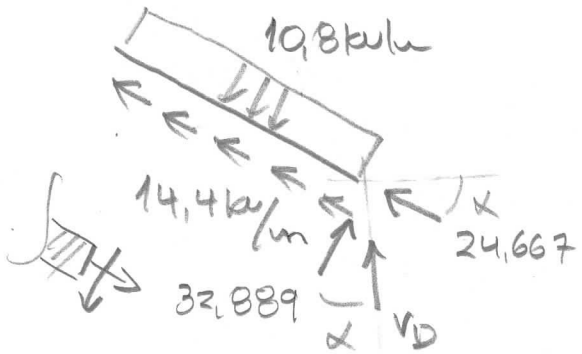
$M_{BC\ MEIO} = -50\times 2{,}5 - 30\times 3^2/2 + 41{,}111\times 6{,}5$

$= \underline{7{,}2215\ kN\,m}$ (Q03)

BARRA CD

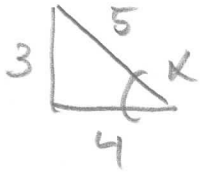
$q_{\perp} = 30\times(3/5)^2 = 10{,}8\ kN/m$

$q_{//} = 30\times 3/5\times 4/5 = 14{,}4\ kN/m$



$V_D = 41{,}111\times 4/5 = 32{,}889\ kN$

$N_D = 41{,}111\times 3/5 = 24{,}667\ kN$



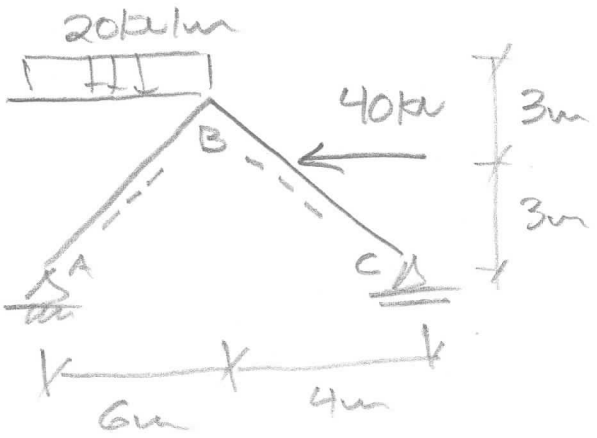
$\cos\alpha = 4/5$

$\operatorname{sen}\alpha = 3/5$

$V_{CD}^{MEIO} = -32{,}889 + 10{,}8\times 2{,}5 = \underline{-5{,}8889\ kN}$ (Q01)

$N_{CD}^{MEIO} = -24{,}667 - 14{,}4\times 2{,}5 = \underline{-60{,}667\ kN}$ (Q02)

B2TE09AND/NC



- E = 200 GPa
- SEÇÃO TRANSVERSAL TUBULAR CIRCULAR DE DIÂMETRO d = 400 mm e ESPESSURA t = 8 mm

PEDE-SE:

Q01) A DEFLEXÃO DO PONTO B (EM mm E COM POSITIVO P/ CIMA)

* CONSIDERAR SOMENTE EFEITOS DE FLEXÃO;

CARREGAMENTO REAL:

$\Sigma M_A = 0 \circlearrowleft (+) \quad V_C \times 10 + 40 \times 3 - 20 \times 6^2/2 = 0$

$V_C = 1/10 (-120 + 10 \times 6^2) = 24\,kN \uparrow$

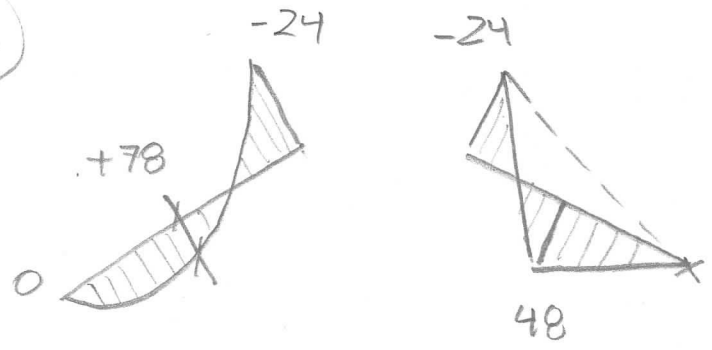
$\Sigma F_V = 0 \quad V_A - 20 \times 6 + 24 = 0 \quad V_A = 96\,kN \uparrow$

$\Sigma F_H = 0 \quad H_A = 40\,kN \rightarrow$

$M_B = 96 \times 6 - 20 \times 6^2/2 - 40 \times 6 = -24\,kN\,m$

$M_B = 24 \times 4 - 40 \times 3 = -24\,kN\,m$

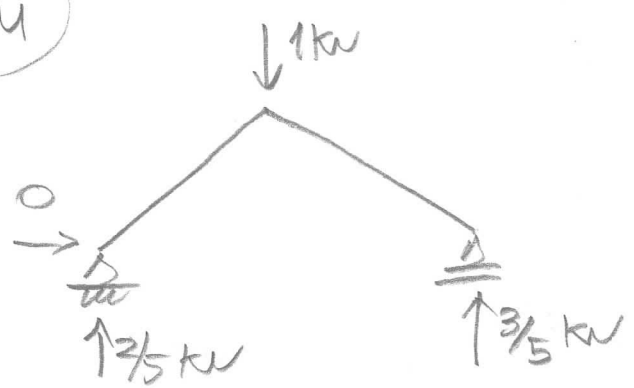
(M)

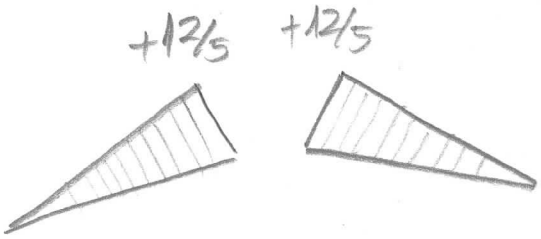


$\frac{-24+0}{2} + \frac{20 \times 6^2}{8} = +78$

$\frac{-24+0}{2} + \frac{40 \times 3 \times 3}{6} = +48$

$\bar{M}$





$\Sigma M_C = 0 \circlearrowleft \oplus \quad V_A \times 10 - 1 \times 4 = 0 \quad V_A = 4/10 = 2/5\ kN \uparrow$

$\Sigma F_V = 0 \uparrow \oplus \quad V_C = 3/5\ kN \uparrow$

$\bar{M}_{IB} = 2/5 \times 6 = 12/5\ kNm$

$\bar{M}_{IB} = 3/5 \times 4 = 12/5\ kNm$

$I = \pi \times 0{,}4^4/64 - \pi \times (0{,}4 - 0{,}016)^4/64 = 1{,}89317 \times 10^{-4}\ m^4$

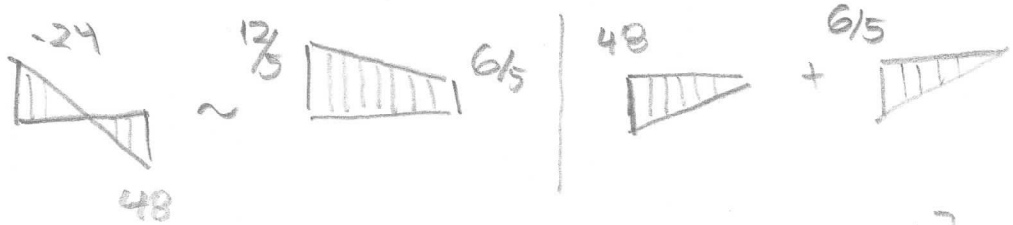
$EI = 200 \times 10^6\ kN/m^2 \times 1{,}89317 \times 10^{-4}\ m^4 = 37863\ kNm^2$

$L_{AB} = \sqrt{6^2+6^2} = 6\sqrt{2}$

$L_{BC} = \sqrt{6^2+4^2} = 2\sqrt{13}$

- BARRA AB $= 1/6 \cdot 6\sqrt{2} \cdot 12/5\,(2 \times 78 - 24) = \underline{448{,}02}$

BARRA BC $=$



$= 1/6\sqrt{13}\,[-24(2 \times 12/5 + 6/5) + 48(12/5 + 2 \times 6/5)] +$

$1/3 \times \sqrt{13} \times 48 \times 6/5 = \underline{121{,}15}$

$\delta_B = \dfrac{448{,}02 + 121{,}15}{37863} = 1{,}5032 \times 10^{-2}\ m$ $\boxed{-15{,}032\ mm}$