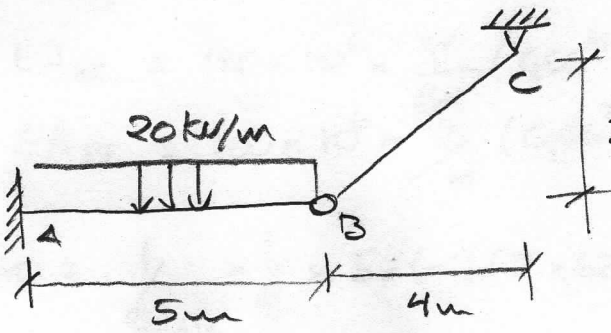
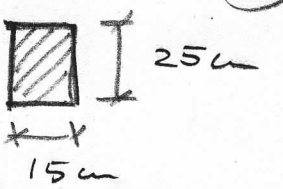
B3TE13AMA

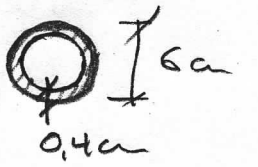


- Barra AB



E = 15000MPa

- Barra BC

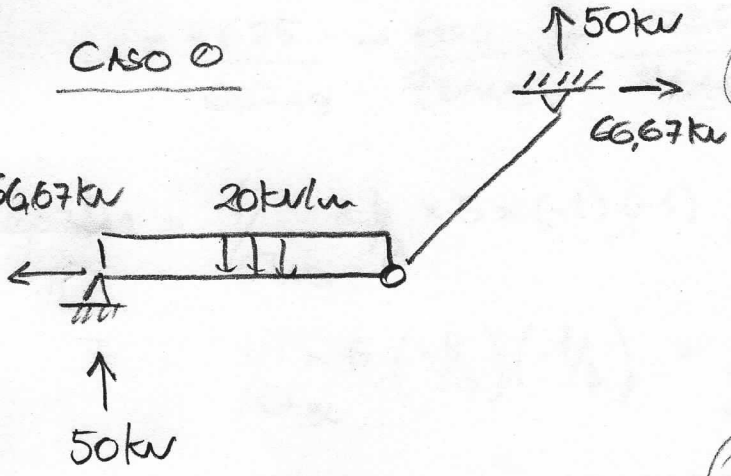


E = 100.000 MPa

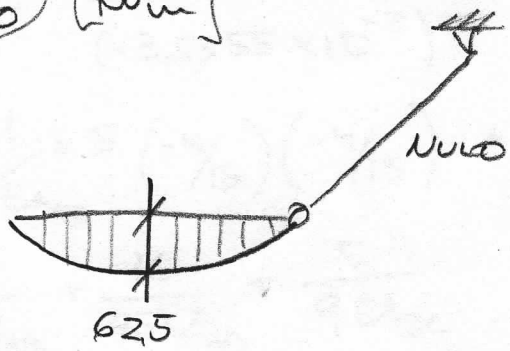
CONSIDERAR COMO REDUNDANTE HIPERESTÁTICA MA ↺;

Q01) ϕ_{10} Q02) f_{A1} Q03) N_{BC}

CASO 0



M_0 [kNm]

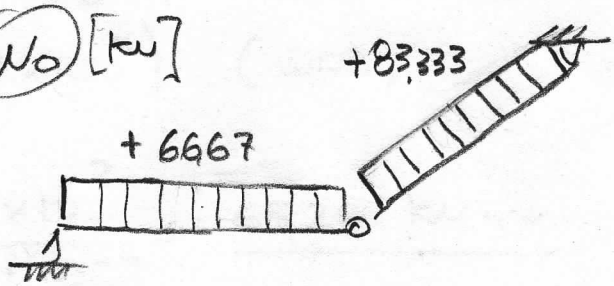


$\Sigma M_C = 0$ ↺+

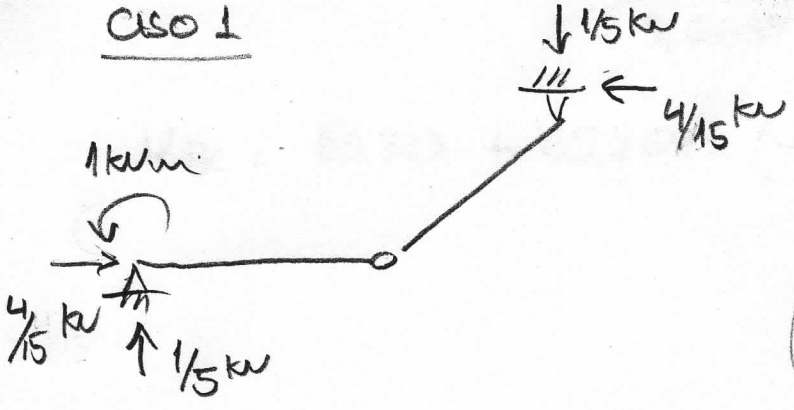
$50 \times 9 - 20 \times 5 \times 6{,}5 - H_A \times 3 = 0$

$H_A = -66{,}67$ kN

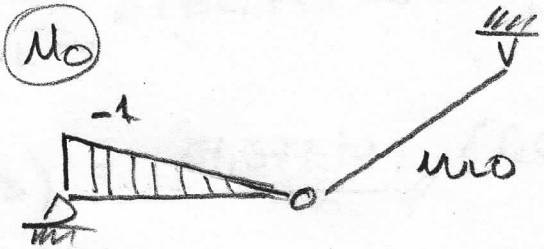
N_0 [kN]



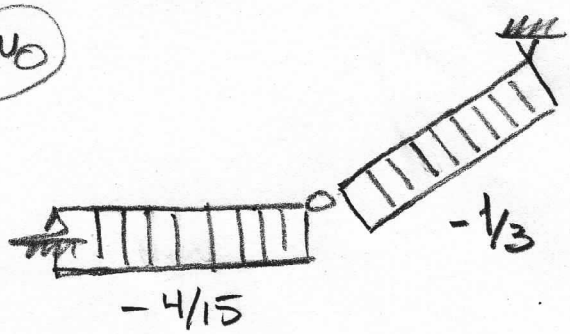
CASO 1



M_0



N_0



$\Sigma M_B = 0$ ↺+ $\frac{1}{5} \times 4 - H_C \times 3 = 0$

$H_C = \frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$

$EI_{AB} = 15\times10^6 \times 0{,}15 \times 0{,}25^3/12 = 2929{,}7\ kN m^2$

$EA_{AB} = 15\times10^6 \times 0{,}15 \times 0{,}25 = 562500\ kN$

$EI_{BC} = 100\times10^6 \times \frac{\pi}{64}(0{,}06^4 - 0{,}052^4) = 27{,}726\ kN m^2$

$EA_{BC} = 100\times10^6 \times \frac{\pi}{4}(0{,}06^2 - 0{,}052^2) = 70371{,}7\ kN$

$$\phi_{10} = \frac{1}{EI_{AB}} \times \frac{1}{6} \times 5 \times (-1)(2\times 62{,}5 + 0) +$$

$$\frac{1}{EA_{AB}} \times 5 \times 66{,}667 \times (-4/15) + \frac{1}{EA_{BC}} \times 5 \times 83{,}333 \times (-1/3)$$

$$= -\frac{625}{6EI_{AB}} - \frac{800}{9EA_{AB}} - \frac{1250}{9EA_{BC}} = \underline{-3{,}7687\times10^{-2}}\ //\ (-3{,}5555\times10^{-2}) \quad (Q01)$$

$$f_{11} = \frac{1}{EI_{AB}} \times \frac{1}{3} \times 5 \times (-1)(-1) + \frac{1}{EA_{AB}} \times 5 (-4/15)(-4/15) +$$

$$\frac{1}{EA_{BC}} \times 5 (-1/3)(-1/3) = \frac{5}{3EI_{AB}} + \frac{16}{45EA_{AB}} + \frac{5}{9EA_{BC}}$$

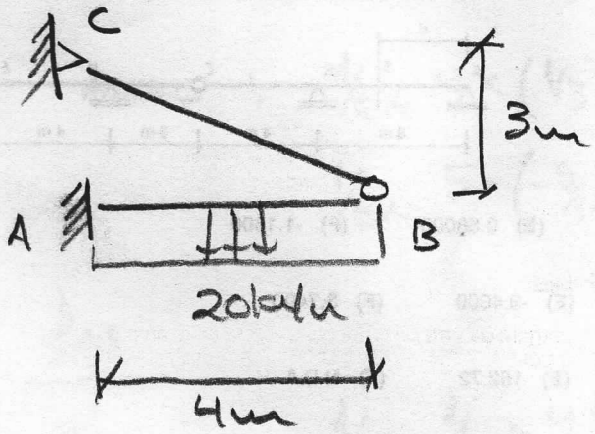
$$= \underline{5{,}7741\times10^{-4}}\ //\ (5{,}6889\times10^{-4}) \quad (Q02)$$

$$M_1 = -\phi_{10} / f_{11} = +\frac{3{,}7687\times10^{-2}}{5{,}7741\times10^{-4}} = \underline{65{,}269\ kN m}$$

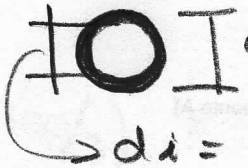
$$\text{ou} \quad = \frac{+3{,}5555\times10^{-2}}{5{,}6889\times10^{-4}} = (62{,}499\ kN m)$$

$$N_{BC} = 83{,}333 + 65{,}269 \times (-1/3) = \underline{61{,}577\ kN}\ // \quad (Q03)$$

B3 TE 13 ANC



$E = 150000\,MPa$



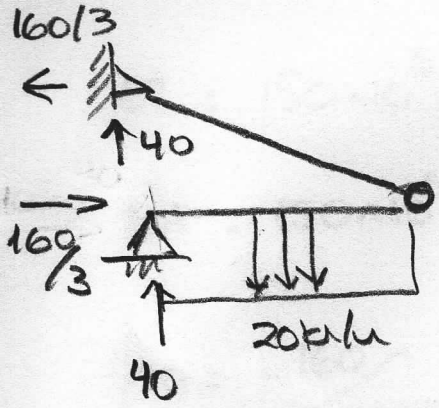
CONSIDERANDO MA COMO REDUNDANTE HIPERESTÁTICA

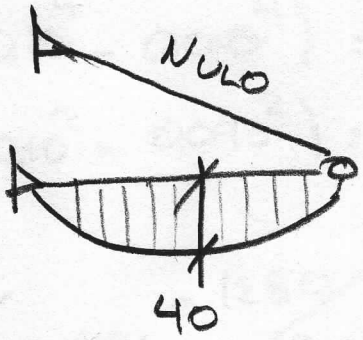
$M_U = \circlearrowleft 1\,kN$

Q01) ϕ_{A0} (rad) Q02) f_{A1} (rad) Q03) N_{AB} [kN]

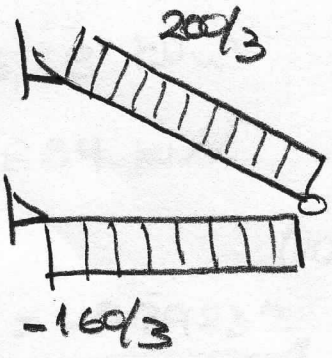
CASO 0

M_0





N_0



$\Sigma M_B = 0 \quad \circlearrowleft +$

$40 \times 4 - H_C \times 3 = 0$

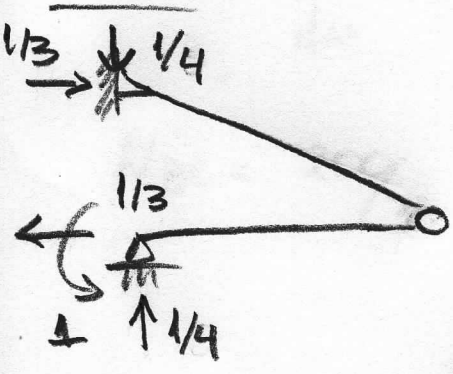
$H_C = 160/3$

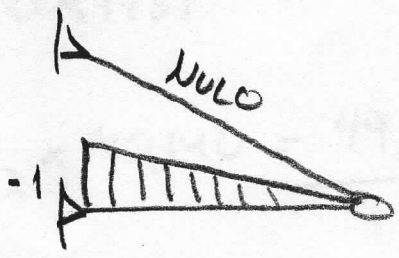
$N_{BC} = \sqrt{40^2 + (160/3)^2}$

$N_{BC} = 200/3$

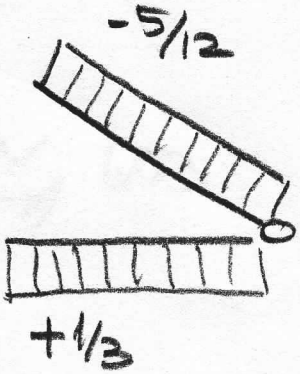
CASO 1

M_0





N_0



$N_{BC} = \sqrt{(1/3)^2 + (1/4)^2} = -5/12$

ϕ_{A0}

FLEXÃO : $\frac{1}{EI}\ \frac{1}{6} \times 4 \times (-1) \times (2 \times 40 + 0) = \frac{-160}{3EI}$

AXIAL : $\frac{1}{EA}\ 4 \times (1/3)(-160/3) = -640/9EA$

$\frac{1}{EA}\ 5 \times (-5/12)(200/3) = -1250/9EA$

f_{A1}

FLEXÃO : $\frac{1}{EI}\ \frac{1}{3} \times 4(-1)(-1) = 4/3EI$

AXIAL : $\frac{1}{EA}\ 4\ (1/3)(1/3) = 4/9EA$

$\frac{1}{EA}\ 5\ (-5/12)(-5/12) = 125/144EA$

$EI = 150 \times 10^6 \times \frac{\pi}{64}(0,10^4 - 0,095^4) = 136,58\ kN m^2$

$EA = 150 \times 10^6 \times \frac{\pi}{4}(0,10^2 - 0,095^2) = 114864\ kN$

Q01

$\phi_{10} = \frac{-160}{3 \times 13658} - \frac{640}{9 \times 114864} - \frac{1250}{9 \times 114864} = -0,39232$

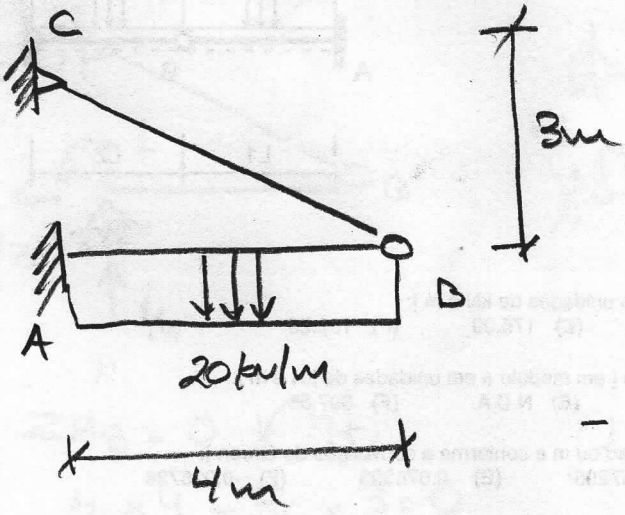
$f_{A1} = \frac{4}{3 \times 13658} + \frac{4}{9 \times 114864} + \frac{125}{144 \times 114864} = 0,0097737$

Q02

$M_A = -\frac{\phi_{10}}{f_{A1}} = \frac{+0,39232}{0,0097737} = 40,140\ kNm$

$N_{BC} = \frac{200}{3} - \frac{5}{12} \times 40,140 = 49,942\ kN$ Q03

B3TE13AND 1POLO



BARRA AB: E = 30000 MPa

40cm
20cm

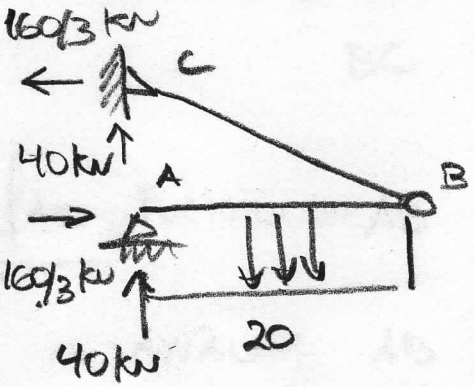
BARRA BC: E = 210000 MPa

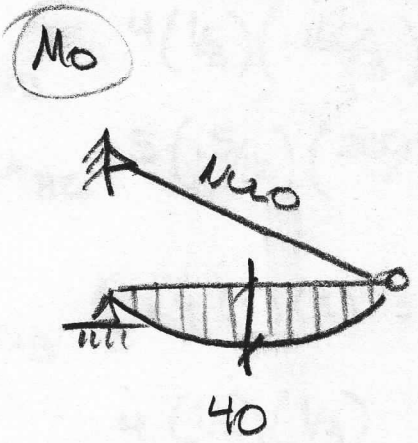
Ø Id = 2cm

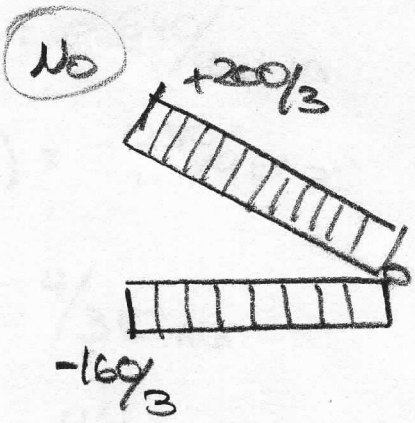
- CONSIDERAR COMO REDUNDANTE HIPERESTÁTICA A REAÇÃO DE MOMENTO FLETOR EM A COM CARGA UNITÁRIA ↶ 1kNm

Q01) ϕ_{A0} [rad] Q02) f_{A1} [rad] Q03) N_{BC} [kN]

CASO 0







$\Sigma M_B = 0$ ↶ ⊕

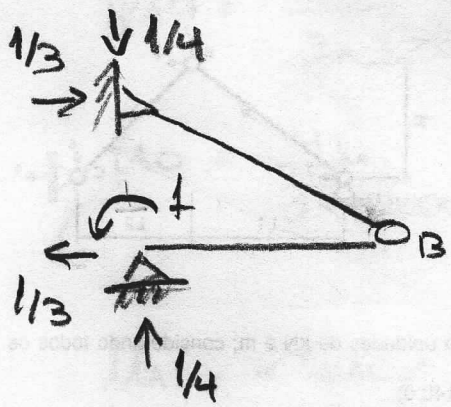
$H_C \times 3 - 40 \times 4 = 0$

$H_C = 160/3 = 53.333$ kN

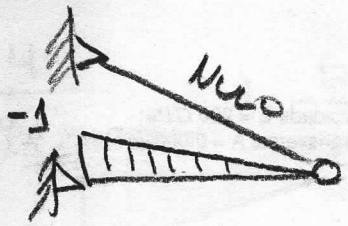
$N_{BC_0} = \sqrt{40^2 + (160/3)^2} = 200/3$

$N_{BC0} = +66.667$ kN

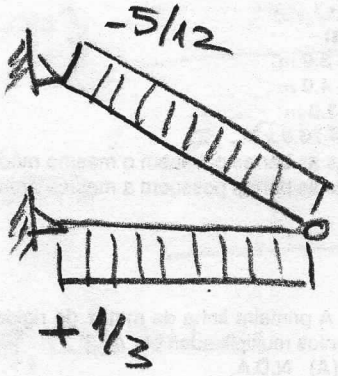
Caso 1



M_1



N_1



$\Sigma M_B = 0 \quad \curvearrowleft (+)$

$1/4 \times 4 - H_C \times 3 = 0$

$H_C = 1/3$

$N_{BC1} = \sqrt{(1/4)^2 + (1/3)^2}$

$N_{BC1} = -5/12$

Q10 FLEXÃO: AB $\frac{1}{EI_{AB}} \; -\frac{1}{6} 4 \times 1 (2 \times 40 + 0) = -160/3 \, EI_{AB}$

AXIAL: AB $\frac{1}{EA_{AB}} \; 4(1/3)(-160/3) = -640/9 EA_{AB}$

BC $\frac{1}{EA_{BC}} \; 5(-5/12)(200/3) = -1250/9 EA_{BC}$

f10 FLEXÃO: AB $\frac{1}{EI_{AB}} \; 1/3 \times 4 (1)(1) = 4/3 EI_{AB}$

AXIAL: AB $\frac{1}{EA_{AB}} \; 4(1/3)(1/3) = 4/9 EA_{AB}$

BC $\frac{1}{EA_{BC}} \; 5(-5/12)(-5/12) = 125/144 EA_{BC}$

$EI_{AB} = 30 \times 10^6 \times 0{,}2 \times 0{,}4^3/12 = 32000$

$EA_{AB} = 30 \times 10^6 \times 0{,}2 \times 0{,}4 = 2400000$

$EA_{DC} = 210 \times 10^6 \times \frac{\pi}{4} \times 0{,}02^2 = 65973$

$$\phi_{A0} = -\frac{160}{3\times 32000} - \frac{640}{9\times 2400000} - \frac{1250}{9\times 65973} = -3{,}8015\times 10^{-3}\ \text{rad}$$

$$f_{A0} = \frac{4}{3\times 32000} + \frac{4}{9\times 2400000} + \frac{125}{144\times 65973} = 5{,}501\times 10^{-5}\ \text{rad}$$

$$M_A = -\phi_{A0}/f_{A0} = \frac{+3{,}8015\times 10^{-3}}{5{,}501\times 10^{-5}} = 69{,}106\ \text{kN}\cdot\text{m}$$

$$N_{BC} = 200/3 - 5/12 \times 69{,}106 = 37{,}873\ \text{kN}\ (+)$$

Q01) $-0{,}0038015$ rad

Q02) $0{,}00005501$ rad

Q03) 37,873 kN