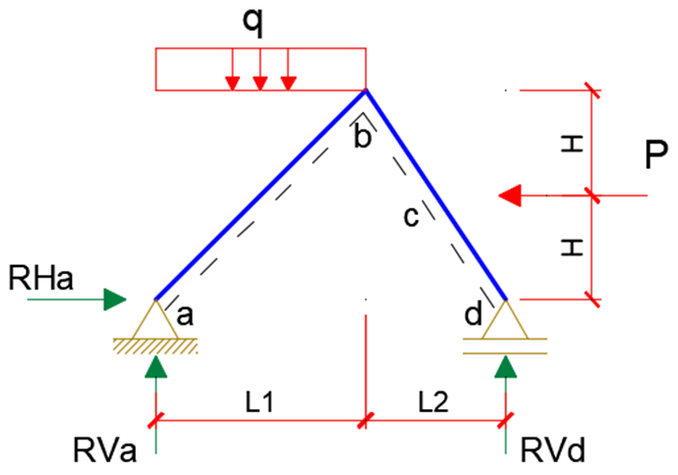
B2T4M0615A

- Considerando somente os efeitos de momentos fletores calcular o deslocamento vertical do ponto "b".



$L1 := 6,0$ m $\quad q := 20,0$ kN/m

$L2 := 4,0$ m $\quad P := 40$ kN

$H := 3,0$ m

- Todas as barras tem seção transversal tubular com diâmetro externo "D" e espessura "t":

$D := 400$ mm

$t := 8$ mm

- Módulo de elasticidade de todas barras:

$E := 200$ GPa

Q01 - Qual o valor do momento fletor no ponto "b" do caso real de carregamento [em "kNm" e sinal conforme a convenção de diagramas] ?
Q02 - Qual o valor do momento fletor no ponto "b" do caso virtual de carregamento empregado para calcular o deslocamento solicitado [em "kNm" e em módulo] ?
Q03 - Qual o valor do deslocamento vertical do ponto "b" [em "mm" e com positivo para cima] ?

Transformação de unidades e propriedades geométricas e do material

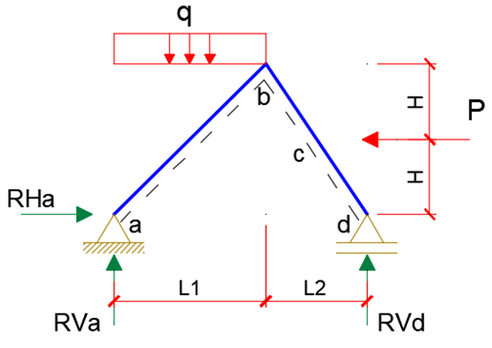
$E := E \cdot 10^{6} = 2 \cdot 10^{8}$ kN m^{2} $\quad D := \frac{D}{1000} = 0,4$ m $\quad t := \frac{t}{1000} = 0,008$ m

$d := D - 2 \cdot t = 0,3840$ m

$Iext := \frac{\pi \cdot D^{4}}{64} = 0,001257$ $\quad Iint := \frac{\pi \cdot d^{4}}{64} = 0,001067$

$I := Iext - Iint = 0,0001893$ m^{4}

Carregamento Real



Reações de Apoio:

ΣFh = 0 (+) para a direita

$RHa - P = 0$ $\quad RHa := P = 40$ kN

ΣMa = 0 (+) antihorário

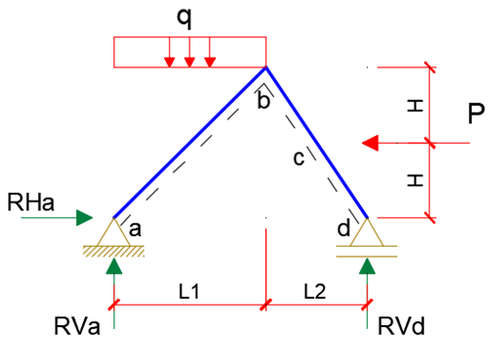
$RVd \cdot (L1 + L2) - q \cdot L1 \cdot \frac{L1}{2} + P \cdot H = 0$

$RVd := \frac{1}{L1 + L2} \cdot \left(\frac{q \cdot L1^{2}}{2} - P \cdot H \right) = 24$ kN

ΣFv = 0 (+) para cima

$RVa - q \cdot L1 + RVd = 0$

$RVa := q \cdot L1 - RVd = 96$ kN



$Mb := RVa \cdot L1 - RHa \cdot 2 \cdot H - q \cdot \frac{L1^2}{2} = -24$ kN m

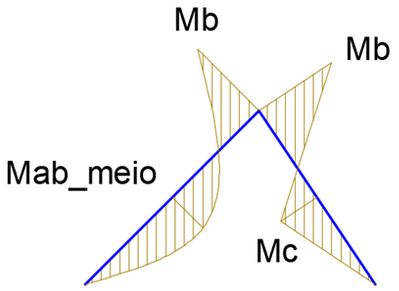
$Mb := RVd \cdot L2 - P \cdot H = -24$ kN m $\qquad Ma := 0$

$Mab_meio := \frac{(Ma + Mb)}{2} + \frac{q \cdot L1^2}{8} = 78$ kN m

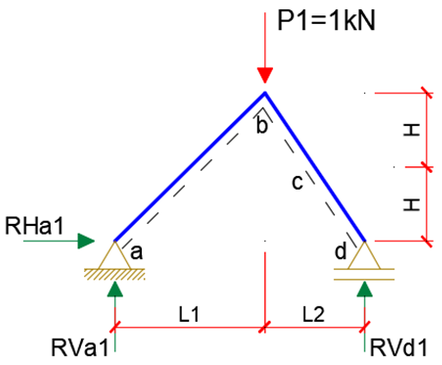
$Md := 0$

$Mc := \frac{(Md + Md)}{2} + \frac{P \cdot H \cdot H}{2 \cdot H} = 60$ kN m

M Real



Carregamento Virtual



Reações de Apoio: $\qquad P1 := 1$ kN

ΣFh = 0 (+) para a direita

$RHa1 := 0$

ΣMa = 0 (+) antihorário

$RVd1 \cdot (L1 + L2) - P1 \cdot L1 = 0$

$RVd1 := \frac{P1 \cdot L1}{L1 + L2} = 0,6$ kN

ΣFv = 0 (+) para cima

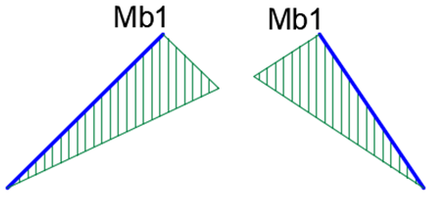
$RVa1 - P1 + RVd1 = 0$

$RVa1 := P1 - RVd1 = 0,4$ kN

$Mb1 := RVa1 \cdot L1 - RHa1 \cdot 2 \cdot H = 2,4$ kN m

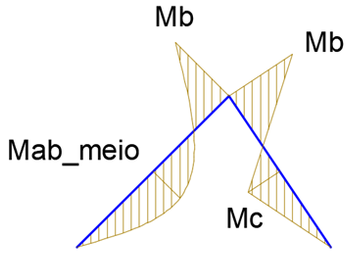
$Mb1 := RVd1 \cdot L2 = 2,4$ kN m

M Virtual

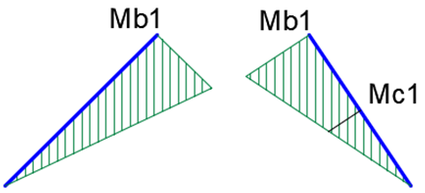


Cálculo do deslocamento vertical do ponto "b":

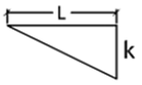
M Real

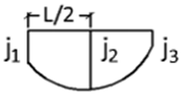


M Virtual



Barra "ab":





1/6 Lk(2j₂+j₃)

$Lab := \sqrt{L1^{2} + (2 \cdot H)^{2}} = 8,4853$

$L := Lab = 8,4853$

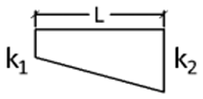
$k := Mb1 = 2,4$

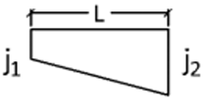
$j2 := Mab_meio = 78$

$j3 := Mb = -24$

$\delta vb_ab := \frac{1}{6} \cdot L \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{E \cdot I} = 0,01183$

Barra "bc":





1/6 L[j₁(2k₁+k₂) + j₂(k₁+2k₂)]

$Lbd := \sqrt{L2^{2} + (2 \cdot H)^{2}} = 7,2111$

$Lbc := \frac{Lbd}{2} = 3,6056$

$Mc1 := \frac{Mb1}{2} = 1,2$

$L := Lbc = 3,6056$

$k1 := Mb1 = 2,4$

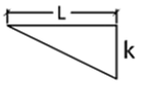
$k2 := Mc1 = 1,2$

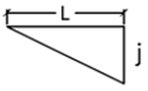
$j1 := Mb = -24$

$j2 := Mc = 60$

$\delta vb_bc := \frac{1}{6} \cdot L \cdot (j1 \cdot (2 \cdot k1 + k2) + j2 \cdot (k1 + 2 \cdot k2)) \cdot \frac{1}{E \cdot I} = 0,0022854$

Barra "cd":





1/3 Ljk

$Lcd := Lbc = 3,6056$

$k := Mc1 = 1,200$

$j := Mc = 60,00$

$L := Lcd = 3,6056$

$\delta vb_cd := \frac{1}{3} \cdot L \cdot j \cdot k \cdot \frac{1}{E \cdot I} = 0,0022854$

$\delta vb := (\delta vb_ab + \delta vb_bc + \delta vb_cd) \cdot (-1000) = -16,4034$ mm

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$Q01 := Mb = -24$

$Q02 := |Mb1| = 2,4$

$Q03 := \delta vb = -16,4034$
