B2T4N0519B

Considerando o modelo estrutural esboçado, responda os questionamentos.

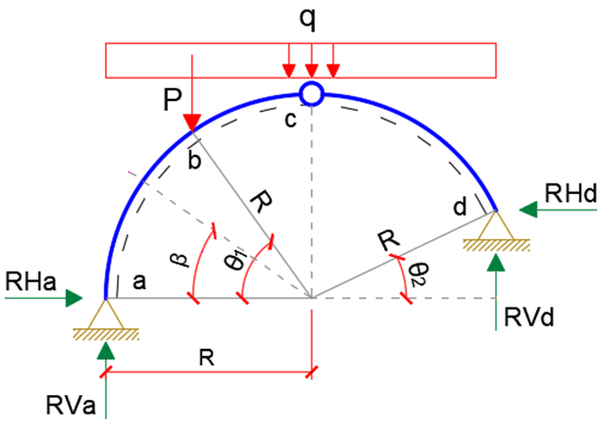
$R := 4\ \text{m}$

$\theta 1 := 50\ °$

$\theta 2 := 20\ °$

$P := 120\ \text{kN}$

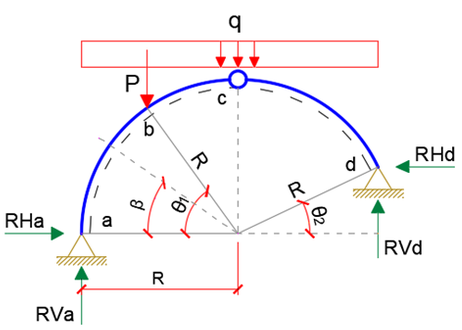
$q := 30\ \frac{\text{kN}}{\text{m}}$



OBS: Todas as respostas devem ser dadas com sinal respeitando a convenção de diagramas
Q01) Qual o valor do esforço cortante à esquerda do ponto "b" ? [em kN]
Q02) Qual o valor do esforço axial à esquerda do ponto "b"? [em kN]
Q03) Qual o valor do esforço cortante à direita do ponto "b"? [em kN]
Q04) Qual o valor do esforço axial à direita do ponto "b"? [em kN]
Q05) Qual o valor do momento fletor em β = 40°? [em kNm]
Q06) Qual o valor do momento fletor em β = 120°? [em kNm]

DICA : Manter a origem do sistema sempre no ponto "a" - não reiniciar a origem para cada trecho de análise.

Reações de Apoio e Esforço na Barra :



- Para cálculo das reações de apoio é necessário compor um sistema de equações com 2 equações e 2 incógnitas;
- OBS: aplicar 2 somatórios de momentos nulos que contemplem sempre as mesmas incógnicas;

$Lad := R + R \cdot \cos(\theta 2) = 7,7588\ \text{m}$

$hd := R \cdot \sin(\theta 2) = 1,3681\ \text{m}$

$Lbd := R \cdot \cos(\theta 1) + R \cdot \cos(\theta 2) = 6,3299\ \text{m}$

$Lbc := R \cdot \cos(\theta 1) = 2,5712\ \text{m}$

ΣMd = 0 (+) horário

$$RVa \cdot Lad - RHa \cdot hd - P \cdot Lbd - q \cdot \frac{Lad^2}{2} = 0$$

$$RVa \cdot Lad - RHa \cdot hd = P \cdot Lbd + q \cdot \frac{Lad^2}{2}$$ Equação 1

ΣMc = 0 (+) horário - olhando para "a"

$$RVa \cdot R - RHa \cdot R - P \cdot Lbc - q \cdot \frac{R^2}{2} = 0$$

$$RVa \cdot R - RHa \cdot R = P \cdot Lbc + q \cdot \frac{R^2}{2}$$ Equação 2

$RVa \cdot Lad - RHa \cdot hd = P \cdot Lbd + q \cdot \frac{Lad^2}{2}$ Equação 1

$RVa \cdot R - RHa \cdot R = P \cdot Lbc + q \cdot \frac{R^2}{2}$ Equação 2

Sistema Matricial de Equações:

[A] . {X} = { B } ===> {X} = [A]^-1 * {B}

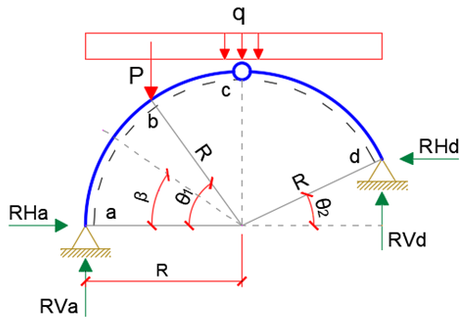
$A := \begin{bmatrix} Lad & -hd \\ R & -R \end{bmatrix} = \begin{bmatrix} 7,7588\ m & -1,3681\ m \\ 4\ m & -4\ m \end{bmatrix}$

$B := \begin{bmatrix} P \cdot Lbd + q \cdot \frac{Lad^2}{2} \\ P \cdot Lbc + q \cdot \frac{R^2}{2} \end{bmatrix} = \begin{bmatrix} 1662,5683 \\ 548,5381 \end{bmatrix}$ kN m

$X := A^{-1} \cdot B = \begin{bmatrix} 230,7978 \\ 93,6633 \end{bmatrix}$ kN

$RVa := X_1 = 230,80$ kN

$RHa := X_2 = 93,663$ kN



ΣFv = 0 (+) para cima

$RVa + RVd - P - q \cdot Lad = 0$

$RVd := -RVa + P + q \cdot Lad = 121,97$ kN

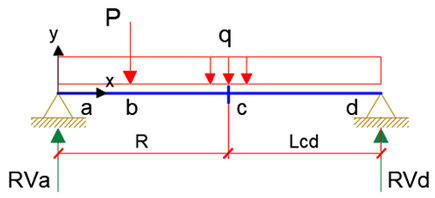
ΣFh = 0 (+) para direita

$RHa - RHd = 0$

$RHd := RHa = 93,663$ kN

Equacionamento dos Esforços Internos:

Viga Análoga



$Lcd := R \cdot \cos(\theta 2) = 3,7588$ m

$Lab := R - R \cdot \cos(\theta 1) = 1,4288$ m

$x := R - R \cdot \cos(\beta)$

$y := R \cdot \sin(\beta)$

$Mabo(\beta) := RVa \cdot x - \frac{q \cdot x^2}{2}$

$Vabo(\beta) := RVa - q \cdot x$

$Mbdo(\beta) := RVa \cdot x - \frac{q \cdot x^2}{2} - P \cdot (x - Lab)$

$Vbdo(\beta) := RVa - q \cdot x - P$

Equação de Momentos Fletores:

$Mab(\beta) := Mabo(\beta) - RHa \cdot y$

$Mbd(\beta) := Mbdo(\beta) - RHa \cdot y$

$M(\beta) := \text{if } \beta \le \theta 1 \text{ } Mab(\beta) \text{ else } Mbd(\beta)$

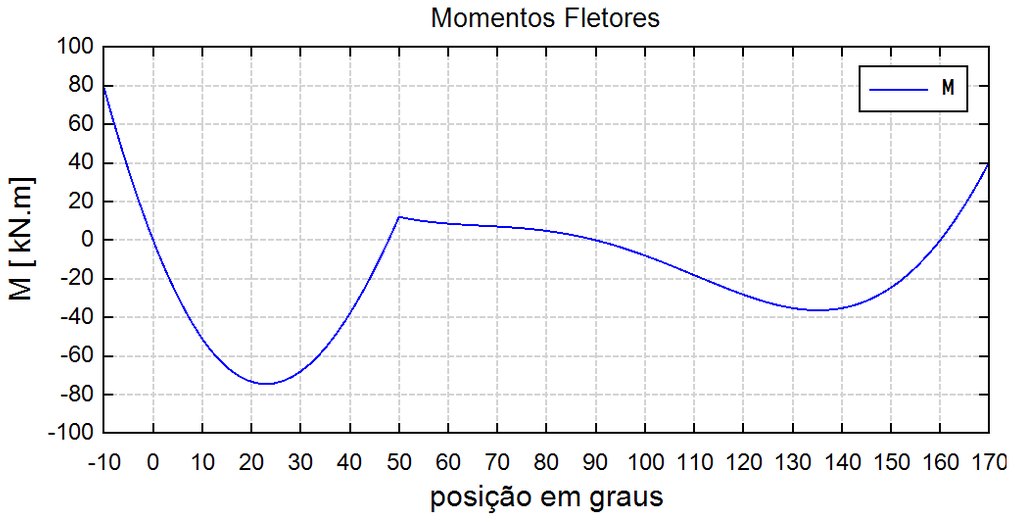
$M(0°) = 0$

$M(90°) = -1,5244 \cdot 10^{-12}$ kN m

$M(160°) = -3,4693 \cdot 10^{-12}$ kN m

$Q05 := M(40°) = -37,973$ kN m

$Q06 := M(120°) = -28,210$ kN m



$\{M(\blacksquare\ °)\cdot 0{,}001$

$\alpha(\beta) := 90\ ° - \beta$

$V(\alpha) = \Sigma VER.\cos\alpha - \Sigma HOR.\mathrm{sen}\alpha$

$N(\alpha) = -\Sigma VER.\mathrm{sen}\alpha - \Sigma HOR.\cos\alpha$

$\Sigma VERab := Vabo(\beta)$ $\Sigma HORab := RHa$

$\Sigma VERbd := Vbdo(\beta)$ $\Sigma HORbd := RHa$

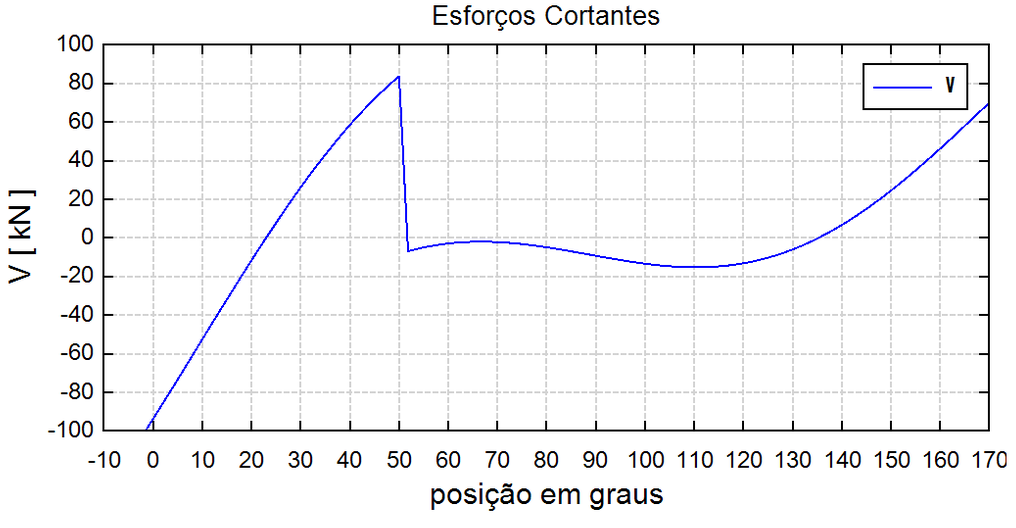
$Vab(\beta) := \Sigma VERab \cdot \cos(\alpha(\beta)) - \Sigma HORab \cdot \sin(\alpha(\beta))$

$Vbd(\beta) := \Sigma VERbd \cdot \cos(\alpha(\beta)) - \Sigma HORbd \cdot \sin(\alpha(\beta))$

$V(\beta) := \text{if } \beta \le \theta 1$
$\quad Vab(\beta)$
$\quad \text{else}$
$\quad\quad Vbd(\beta)$

$Q01 := Vab(50\ °) = 83{,}7589\ \text{kN}$

$Q03 := Vbd(50\ °) = -8{,}1664\ \text{kN}$



$\{V(\blacksquare\ °)\cdot 0{,}001$

$V(\alpha) = \Sigma VER.\cos\alpha - \Sigma HOR.\mathrm{sen}\alpha$

$N(\alpha) = -\Sigma VER.\mathrm{sen}\alpha - \Sigma HOR.\cos\alpha$

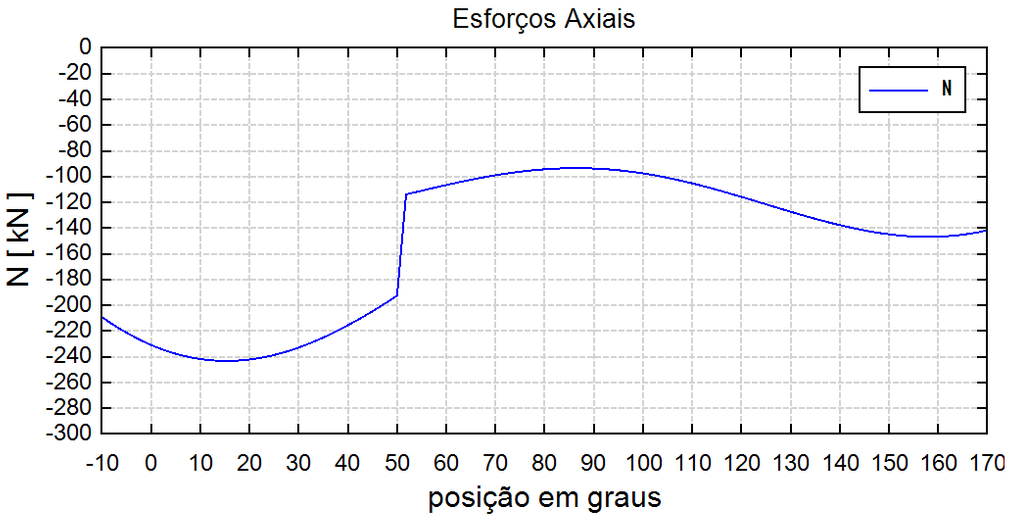
$Nab(\beta) := (-\Sigma VERab)\cdot\sin(\alpha(\beta)) - \Sigma HORab\cdot\cos(\alpha(\beta))$

$Nbd(\beta) := (-\Sigma VERbd)\cdot\sin(\alpha(\beta)) - \Sigma HORbd\cdot\cos(\alpha(\beta))$

$N(\beta) := \text{if } \beta \le \theta 1$
$\quad Nab(\beta)$
$\quad \text{else}$
$\quad Nbd(\beta)$

$Q02 := Nab(50\ °) = -192{,}55\ \text{kN}$

$Q04 := Nbd(50\ °) = -115{,}42\ \text{kN}$



$\{N(\blacksquare\ °) \cdot 0{,}001$

$Q01 = 83{,}759\ \text{kN}$

$Q02 = -192{,}55\ \text{kN}$

$Q03 = -8{,}1664\ \text{kN}$

$Q04 = -115{,}42\ \text{kN}$

$Q05 = -37{,}973\ \text{kN m}$

$Q06 = -28{,}210\ \text{kN m}$
