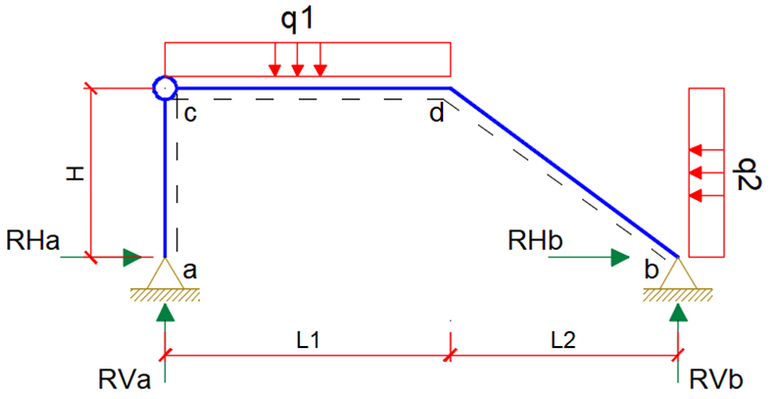
B2T4N0609B

- Considerando somente os efeitos de momentos fletores calcular o deslocamento vertical e a rotação do ponto "d".



$L1 := 4,0$ m $\quad q1 := 25,0$ kN/m

$L2 := 4,0$ m $\quad q2 := 10,0$ kN/m

$H := 3,0$ m

- Todas as barras tem seção transversal retangular com dimensões:

$b := 0,20$ m base (entrante no plano do papel)

$h := 0,40$ m altura (assente ao plano do papel)

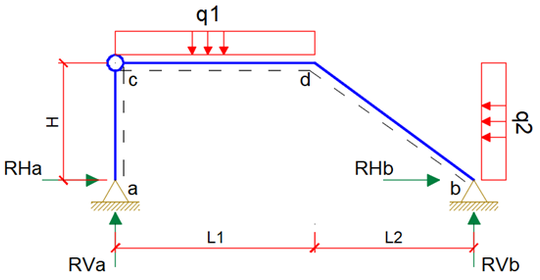
- Módulo de elasticidade para todas as barras:

$E := 15 \cdot 10^{6}$ kN/m²

Q01 - Qual o valor do deslocamento vertical do ponto "d" [em "mm" e positivo para cima] ?

Q02 - Qual o valor da rotação do ponto "d" [em "rad" e positivo para antihorário] ?

Carregamento Real



Reações de Apoio:

ΣMc = 0 olhando para "a" (+) giro horário

$RHa := 0$ kN

ΣFh = 0 (+) para direita

$RHb - q2 \cdot H = 0 \qquad RHb := q2 \cdot H = 30$ kN

ΣMa = 0 (+) giro antihorário

$RVb \cdot (L1 + L2) - \frac{q1 \cdot L1^{2}}{2} + \frac{q2 \cdot H^{2}}{2} = 0 \qquad RVb := \frac{1}{L1 + L2} \cdot \left(\frac{q1 \cdot L1^{2}}{2} - \frac{q2 \cdot H^{2}}{2} \right) = 19,375$ kN

ΣFv = 0 (+) para cima

$RVa + RVb - q1 \cdot L1 = 0 \qquad RVa := -RVb + q1 \cdot L1 = 80,625$ kN

Equações de Momentos Fletores do Carregamento Real:

$Mac(x) := 0$ barra tipo escora-tirante

$Mcd(x) := RVa \cdot x - \frac{q1 \cdot x^{2}}{2} = \frac{645}{8} \cdot x - \frac{25 \cdot x^{2}}{2} \qquad Md := Mcd(L1) = 122,5 \qquad Mcd_meio := Mcd\left(\frac{L1}{2}\right) = 111,25$

convertendo q2 em carga perpendicular e paralela:

$Ldb := \sqrt{H^{2} + L2^{2}} = 5 \qquad sen\alpha := \frac{H}{Ldb} = 0,6$

$cos\alpha := \frac{L1}{Ldb} = 0,8$

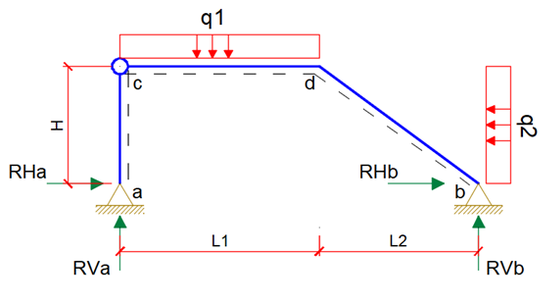
$q2_perp := q2 \cdot (sen\alpha)^{2} = 3,6 \ \frac{kN}{m}$

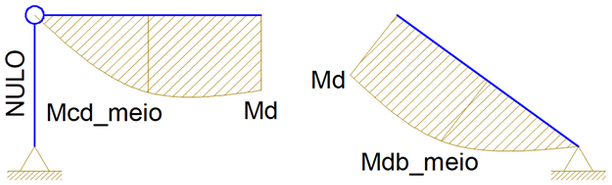
$VERT_d := RVa - q1 \cdot L1 = -19,375$ kN

$Vd := VERT_d \cdot cos\alpha = -15,5$ kN

$Mdb(x) := Md + Vd \cdot x - \frac{q2_perp \cdot x^{2}}{2} = \frac{245}{2} + \left(-\frac{31}{2}\right) \cdot x - \frac{\frac{18}{5} \cdot x^{2}}{2} \qquad Mdb\left(\frac{Ldb}{2}\right) = 72,5$

Traçado do Diagrama e Momentos Fletores do Carregamento Real





$Md := RVa \cdot L1 - \frac{q1 \cdot L1^{2}}{2} = 122,5 \quad kN\ m$

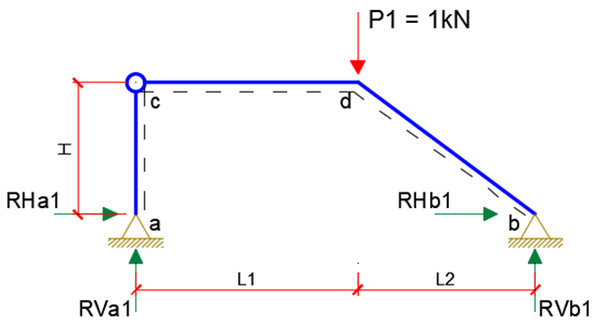
$Md := RVb \cdot L2 + RHb \cdot H - \frac{q2 \cdot H^{2}}{2} = 122,5 \quad kN\ m$

$Mc := 0 \qquad Mb := 0$

$Mcd_meio := \frac{(Mc + Md)}{2} + \frac{q1 \cdot L1^{2}}{8} = 111,25 \quad kN\ m$

$Mdb_meio := \frac{(Md + Mb)}{2} + \frac{q2 \cdot H^{2}}{8} = 72,5 \quad kN\ m$

Carregamento Virtual para cálculo do deslocamento de translação vertical em "d":



Reações de Apoio: $P1 := 1$

ΣMc = 0 olhando para "a" (+) giro horário

$RHa1 := 0 \quad kN$

ΣFh = 0 (+) para direita

$RHa1 + RHb1 = 0 \qquad RHb1 := -RHa1 = 0\ kN$

ΣMa = 0 (+) giro antihorário $\quad RVb1 \cdot (L1 + L2) - P1 \cdot L1 = 0 \qquad RVb1 := \frac{1}{L1 + L2} \cdot (P1 \cdot L1) = 0,5\ kN$

ΣFv = 0 (+) para cima $\quad RVa1 + RVb1 - P1 = 0 \qquad RVa1 := -RVb1 + P1 = 0,5\ kN$

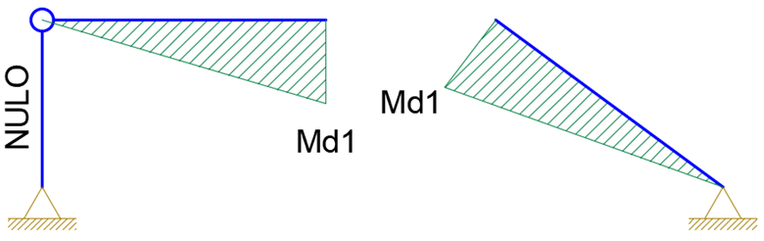
Equações de Momentos Fletores do Carregamento Virtual para Deflexão do ponto "d":

$Mac1(x) := 0$ Barra escora-tirante;

$Mcd1(x) := RVa1 \cdot x = \frac{x}{2} \qquad Md1 := Mcd1(L1) = 2 \quad kN\ m$

$Mdb1(x) := Md1 + (RVa1 - P1) \cdot cos\alpha \cdot x = 2 + \left(\frac{1}{2} - 1\right) \cdot \frac{4}{5} \cdot x \qquad Mdb1(0) = 2$

Traçado do Diagrama e Momentos Fletores do Carregamento Virtual para Deflexão do ponto "d":



$Md1 := RVa1 \cdot L1 = 2$

$Md1 := RVb1 \cdot L2 = 2$

Cálculo do deslocamento vertical no ponto "d":

Via Integração Normal:

$I := \frac{b \cdot h^3}{12} = 0,001067 \ m^4$ $\quad E = 1,5 \cdot 10^7 \ \frac{kN}{m^2}$

barra "cd":

$$\delta vdm_cd := \frac{1}{E \cdot I} \cdot \int_0^{L1} Mcd1(x) \cdot Mcd(x) \, dx = 0,028750$$

barra "db":

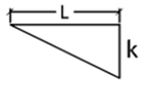
$$\delta vdm_db := \frac{1}{E \cdot I} \cdot \int_0^{Ldb} Mdb1(x) \cdot Mdb(x) \, dx = 0,027865$$

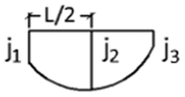
$\delta vd := (\delta vdm_cd + \delta vdm_db) \cdot (-1000) = -56,6146 \ mm$

Via Integração por Tabelas:



Barra "cd":





1/6 Lk(2j₂+j₃)

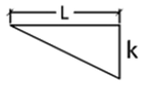
$k := Md1 = 2$

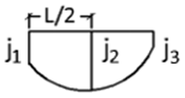
$j2 := Mcd_meio = 111,2$

$j3 := Md = 122,5$

$\delta vdm_cd := \frac{1}{6} \cdot L1 \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{E \cdot I} = 0,02875$

Barra "db":





1/6 Lk(2j₂+j₃)

$k := Md1 = 2,000$ $\quad Ldb = 5$

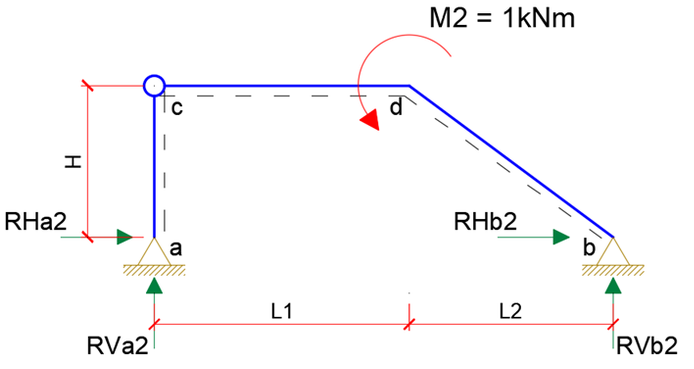
$j2 := Mdb_meio = 72,50$

$j3 := Md = 122,5$

$\delta vdm_db := \frac{1}{6} \cdot Ldb \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{E \cdot I} = 0,027865$

$Q01 := \delta vd = -56,6146$

Carregamento Virtual para cálculo do deslocamento de rotação em "d":



Reações de Apoio: $M2 := 1$

ΣMc = 0 olhando para "a" (+) giro horário

$RHa2 := 0 \ kN$

ΣFh = 0 (+) para direita

$RHa2 + RHb2 = 0$ $\quad RHb2 := -RHa2 = 0 \ kN$

ΣMa = (+) giro antihorário

$RVb2 \cdot (L1 + L2) + M2 = 0$

$RVb2 := -\frac{M2}{L1 + L2} = -0,125 \ kN$

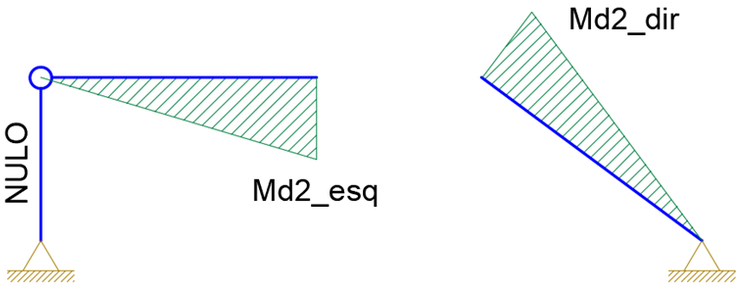
ΣFv = 0 (+) para cima $RVa2 + RVb2 = 0$ $RVa2 := -RVb2 = 0,125\ kN$

Equações de Momentos Fletores do Carregamento Virtual para Rotação do ponto "d":

$Mac2(x) := 0$

$Mcd2(x) := RVa2 \cdot x$ $Md_esq := Mcd2(L1) = 0,5$

$Mdb2(x) := Md_esq - M2 + RVa2 \cdot cos\alpha \cdot x = \frac{1}{2} - 1 + \frac{1}{8} \cdot \frac{4}{5} \cdot x$ $Mdb2(Ldb) = 0$



$Md2_esq := RVa2 \cdot L1 = 0,5\ kN\ m$

$Md2_dir := Md_esq - M2 = -0,5\ kN\ m$

Cálulo da Rotação em "d":

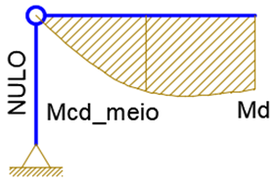
Integração Normal

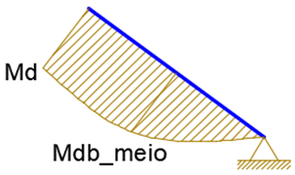
Barra "cd": $\varphi dm_cd := \frac{1}{E \cdot I} \cdot \int_0^{L1} Mcd2(x) \cdot Mcd(x)\, dx = 0,0071875$

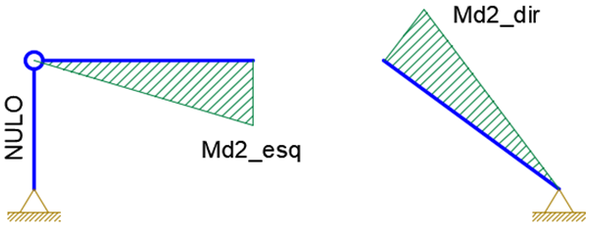
Barra "db": $\varphi dm_db := \frac{1}{E \cdot I} \cdot \int_0^{Ldb} Mdb2(x) \cdot Mdb(x)\, dx = -0,0069661$

$\varphi d := \varphi dm_cd + \varphi dm_db = 0,00022135$ $Q02 := \varphi d = 0,00022135$

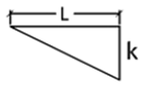
Integração por Tabela

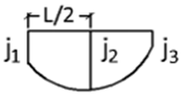






Barra "cd":





1/6 Lk(2j₂+j₃)

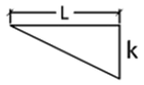
$k := Md2_esq = 0,5000$

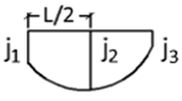
$j2 := Mcd_meio = 111,2$

$j3 := Md = 122,5$

$mcd2 := \frac{1}{6} \cdot L1 \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{E \cdot I} = 0,0071875$

Barra "db":





1/6 Lk(2j₂+j₃)

$k := Md2_dir = -0,5000$

$j2 := Mdb_meio = 72,50$

$j3 := Md = 122,5$

$mdb2 := \frac{1}{6} \cdot Ldb \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{E \cdot I} = -0,0069661$

$Q01 = -56,615$

$Q02 = 0,00022135$
