TE1_E07-02

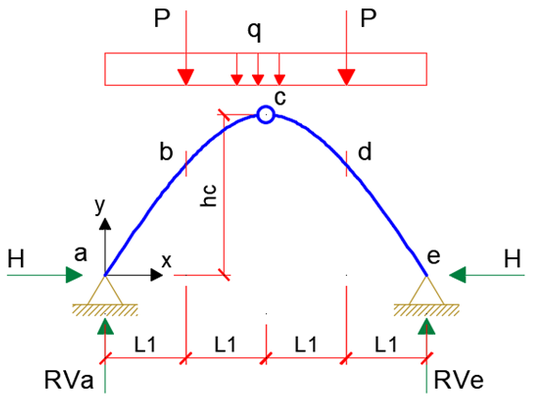
Traçar os diagramas de esforços internos para o arco parabólico de 2° grau indicado.

$L1 := 5\ m$

$hc := 8\ m$

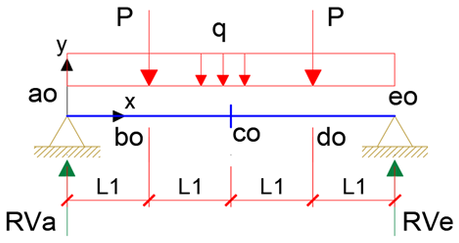
$P := 2\ tf$

$q := 4\ \frac{tf}{m}$



Reações de Apoio e Equações de Esforços na Viga Análoga:

Viga Análoga:



* Simetria carregamento e vinculação.

$RVa = RVe$

$RVa := \frac{1}{2} \cdot (q \cdot 4 \cdot L1 + 2 \cdot P) = 42\ tf$

$Mabo(x) := RVa \cdot x - \frac{q \cdot x^2}{2}$

$Vabo(x) := RVa - q \cdot x$

$Mbdo(x) := RVa \cdot x - \frac{q \cdot x^2}{2} - P \cdot (x - L1)$

$Vbdo(x) := RVa - q \cdot x - P$

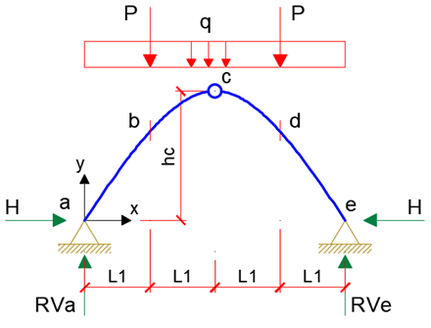
$Mdeo(x) := RVa \cdot x - \frac{q \cdot x^2}{2} - P \cdot (x - L1) - P \cdot (x - 3 \cdot L1)$

$Vdeo(x) := RVa - q \cdot x - P - P$

$Mco := Mbdo(2 \cdot L1) = 210\ tf\ m$

$H := \frac{Mco}{hc} = 26{,}25\ tf$

Equacionamento da Geometria do Arco:



Ponto "a": $xa := 0$ $ya := 0$

Ponto "c": $xc := 2 \cdot L1 = 10\ m$ $yc := hc = 8\ m$

Ponto "e": $xe := 4 \cdot L1 = 20\ m$ $ye := 0$

$y := a + b \cdot x + c \cdot x^2$

$y_a := a + b \cdot xa + c \cdot xa^2$

$y_c := a + b \cdot xc + c \cdot xc^2$

$y_e := a + b \cdot xe + c \cdot xe^2$

Sistema Matricial de Equações:
{ Y } = [X] * { abc }

$$Y := \begin{bmatrix} ya \\ yc \\ ye \end{bmatrix} = \begin{bmatrix} 0 \\ 8\ \mathrm{m} \\ 0 \end{bmatrix} \qquad X := \begin{bmatrix} 1 & xa & xa^2 \\ 1 & xc & xc^2 \\ 1 & xe & xe^2 \end{bmatrix}$$

{ abc } = [X]^-1 * { Y }

$$abc := X^{-1} \cdot Y = \begin{bmatrix} 0 \\ 1,6 \\ -0,08 \cdot \frac{1}{\mathrm{m}} \end{bmatrix}$$

$$a := abc_1 = 0$$

$$b := abc_2 = 1,6$$

$$c := abc_3 = -0,08 \cdot \frac{1}{\mathrm{m}}$$

$$y(x) := a + b \cdot x + c \cdot x^2 = 0 + \frac{8}{5} \cdot x + \left(-\frac{2}{25} \cdot \frac{1}{\mathrm{m}}\right) \cdot x^2$$

$$\alpha(x) := \mathrm{atan}\left(\frac{\mathrm{d}}{\mathrm{d}\,x} y(x)\right)$$

Equacionamento dos Esforços:

Equações diferentes nos três trechos, a saber, trecho "ab", trecho "bd" e trecho "de":

$M_S(x) = M_{S0}(x) - H.y(x)$
$V_S(x) = +V_{S0}(x)\ \cos\alpha(x) - H\ \mathrm{sen}\alpha(x)$
$Ns(x) = -V_{S0}(x)\ \mathrm{sen}\alpha(x) - H\ \cos\alpha(x)$

Momento Fletor:

$$Mab(x) := Mabo(x) - H \cdot y(x)$$

$$Mbd(x) := Mbdo(x) - H \cdot y(x)$$

$$Mde(x) := Mdeo(x) - H \cdot y(x)$$

$$M(x) := \begin{array}{l} \mathrm{if}\ x\ \mathrm{m} < L1 \\ \quad Mab(x\ \mathrm{m}) \\ \mathrm{else} \\ \quad \mathrm{if}\ x\ \mathrm{m} < 3 \cdot L1 \\ \quad\quad Mbd(x\ \mathrm{m}) \\ \quad \mathrm{else} \\ \quad\quad Mde(x\ \mathrm{m}) \end{array}$$

Esforço Cortante:

$$Vab(x) := Vabo(x) \cdot \cos(\alpha(x)) - H \cdot \sin(\alpha(x))$$

$$Vbd(x) := Vbdo(x) \cdot \cos(\alpha(x)) - H \cdot \sin(\alpha(x))$$

$$Vde(x) := Vdeo(x) \cdot \cos(\alpha(x)) - H \cdot \sin(\alpha(x))$$

$$V(x) := \begin{array}{l} \mathrm{if}\ x\ \mathrm{m} < L1 \\ \quad Vab(x\ \mathrm{m}) \\ \mathrm{else} \\ \quad \mathrm{if}\ x\ \mathrm{m} < 3 \cdot L1 \\ \quad\quad Vbd(x\ \mathrm{m}) \\ \quad \mathrm{else} \\ \quad\quad Vde(x\ \mathrm{m}) \end{array}$$

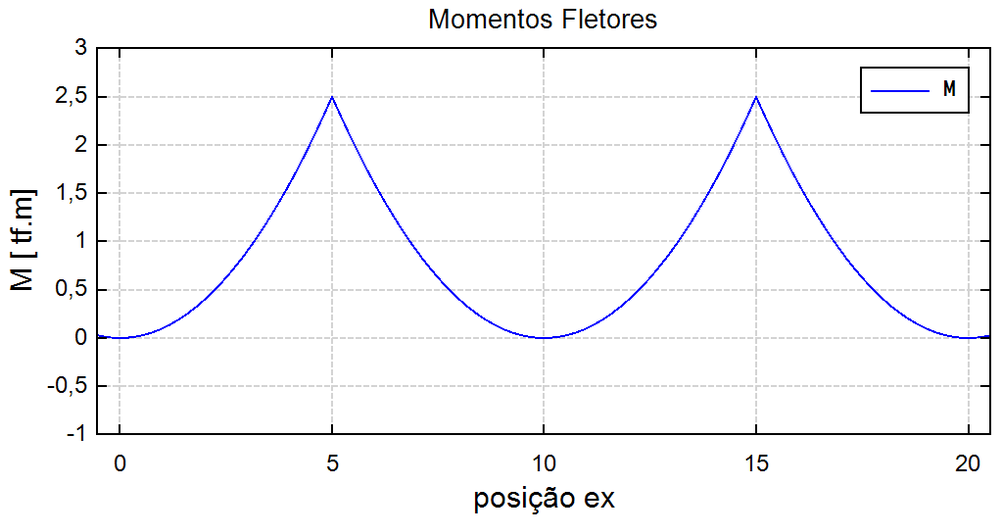
Esforço Axial:

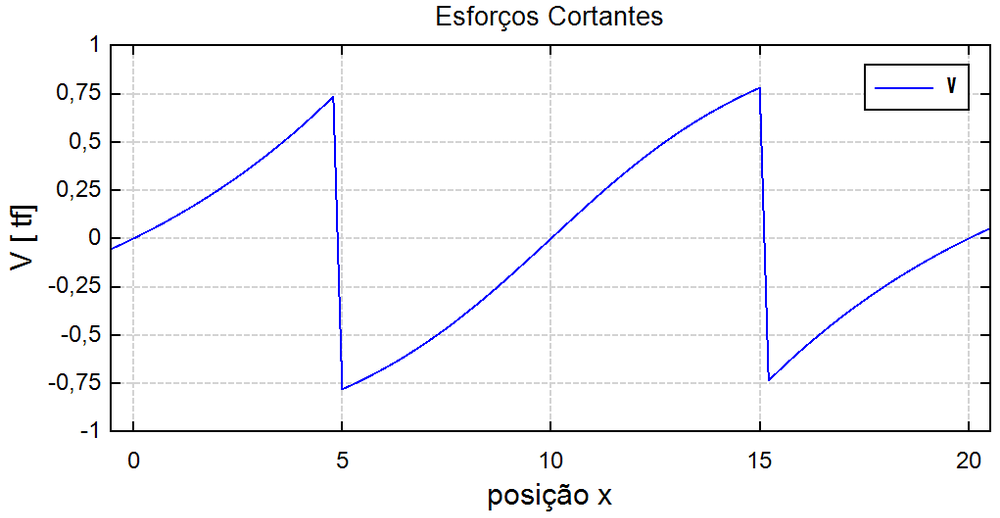
$$Nab(x) := (-Vabo(x)) \cdot \sin(\alpha(x)) - H \cdot \cos(\alpha(x))$$

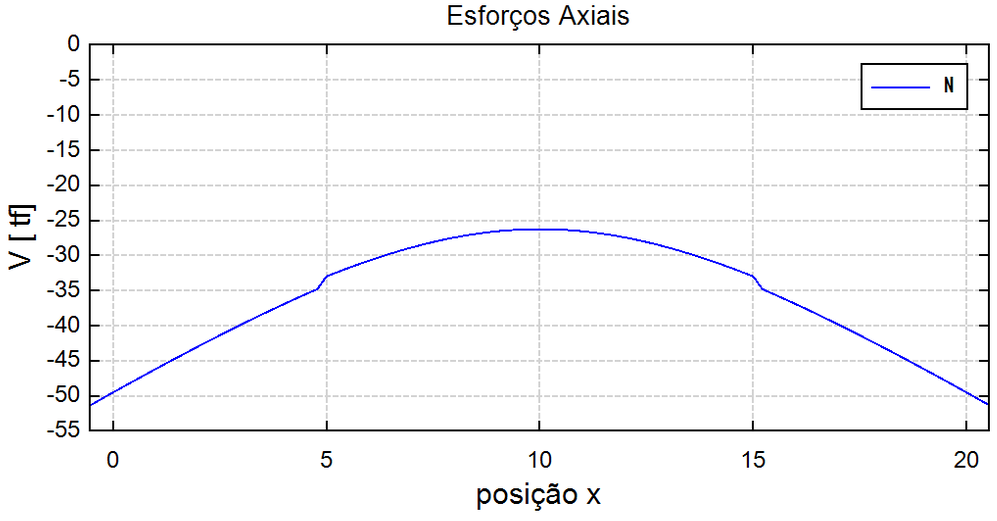
$$Nbd(x) := (-Vbdo(x)) \cdot \sin(\alpha(x)) - H \cdot \cos(\alpha(x))$$

$$Nde(x) := (-Vdeo(x)) \cdot \sin(\alpha(x)) - H \cdot \cos(\alpha(x))$$

$$N(x) := \begin{array}{l} \mathrm{if}\ x\ \mathrm{m} < L1 \\ \quad Nab(x\ \mathrm{m}) \\ \mathrm{else} \\ \quad \mathrm{if}\ x\ \mathrm{m} < 3 \cdot L1 \\ \quad\quad Nbd(x\ \mathrm{m}) \\ \quad \mathrm{else} \\ \quad\quad Nde(x\ \mathrm{m}) \end{array}$$


Momentos Fletores
M
M [tf.m]
posição ex
3
2,5
2
1,5
1
0,5
0
-0,5
-1
0
5
10
15
20


Esforços Cortantes
V
V [tf]
posição x
1
0,75
0,5
0,25
0
-0,25
-0,5
-0,75
-1
0
5
10
15
20


Esforços Axiais
N
V [tf]
posição x
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
-55
0
5
10
15
20