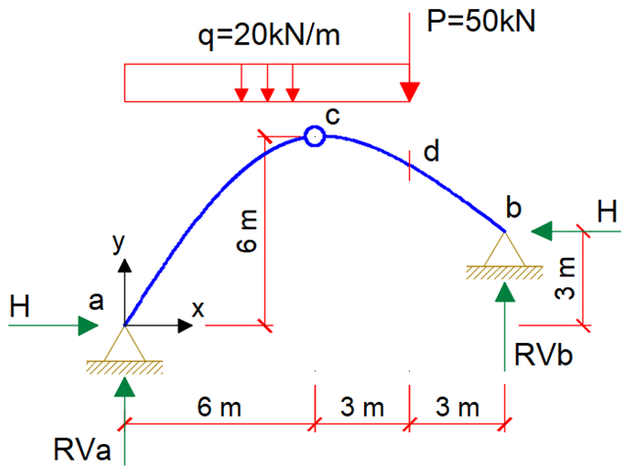
TE1_E07-08

- Traçar os diagramas de momento fletor, esforço cortante e esforço axial para o arco parabólico de segundo grau esboçado, determinando os valores destes esforços internos a cada 1 metro do eixo horizontal (x).



$Lac := 6,0$ $P := 50$

$Lcd := 3,0$ $q := 20$

$Ldb := 3,0$

$f := 6,0$

$Hab := 3,0$

$Lvao := Lac + Lcd + Ldb = 12$

$Lad := Lac + Lcd = 9$

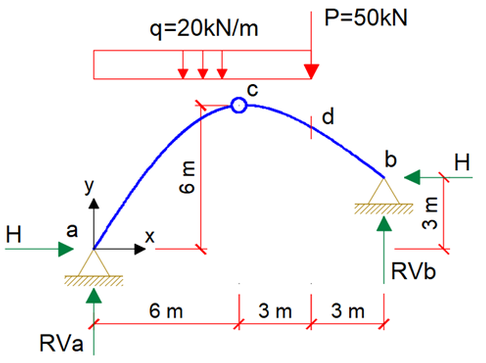
Reações de Apoio:

ΣMc = 0 (+) horário - olhando para "a"

```
RVa.Lac - H.f - q.Lac^2 / 2 = 0
```

ΣMb = 0 (+) horário

```
RVa.Lvao - H.Hab - P.Ldb - q.(Lad).(Lad/2+Ldb) = 0
```



Sistema de equações:

```
RVa.Lac - H.f = q.Lac^2 / 2
RVa.Lvao - H.Hab = P.Lbd + q.(Lad).(Lad/2+Ldb)
```

$$matrizA := \begin{bmatrix} Lac & -f \\ Lvao & -Hab \end{bmatrix} = \begin{bmatrix} 6 & -6 \\ 12 & -3 \end{bmatrix} \qquad vetorB := \begin{bmatrix} \frac{q \cdot Lac^{2}}{2} \\ P \cdot Ldb + q \cdot Lad \cdot \left(\frac{Lad}{2} + Ldb \right) \end{bmatrix} = \begin{bmatrix} 360 \\ 1500 \end{bmatrix}$$

$$vetorX := matrizA^{-1} \cdot vetorB = \begin{bmatrix} 146,67 \\ 86,667 \end{bmatrix}$$

$q \cdot Lad + P = 230$

$RVa := vetorX_{1} = 146,67$

$H := vetorX_{2} = 86,6667$

ΣFh = 0 (+) para direita: H igual em módulo porém com sentido contrário.

ΣFv = 0 (+) para cima

```
RVa + RVb - P - q.Lad = 0
```

$RVb := -RVa + P + q \cdot Lad = 83,333$

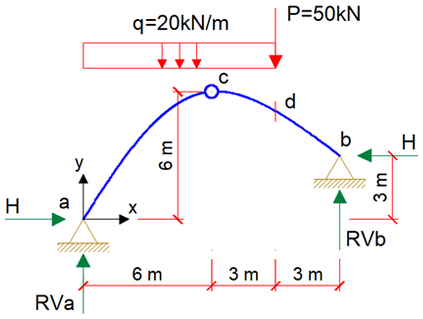
Equação do Arco Parabólico de 2° Grau:

$x1 := 0 \qquad y1 := 0$

$x2 := Lac = 6 \qquad y2 := f = 6$

$x3 := Lvao = 12 \qquad y3 := Hab = 3$

$$matrizA := \begin{bmatrix} 1 & x1 & x1^2 \\ 1 & x2 & x2^2 \\ 1 & x3 & x3^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 6 & 36 \\ 1 & 12 & 144 \end{bmatrix} \qquad vetorB := \begin{bmatrix} y1 \\ y2 \\ y3 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 3 \end{bmatrix}$$



$$vetorX := matrizA^{-1} \cdot vetorB = \begin{bmatrix} 0 \\ 1,75 \\ -0,125 \end{bmatrix}$$

$a := vetorX_1 = 0$

$b := vetorX_2 = 1,75$

$c := vetorX_3 = -0,125$

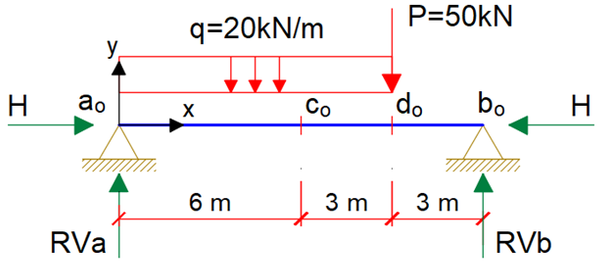
$$y(x) := a + b \cdot x + c \cdot x^2 = 0 + \frac{7}{4} \cdot x + \left(-\frac{1}{8}\right) \cdot x^2$$

Equações da Viga Análoga (esforços verticais considerando somente a variação ao longo do eixo "x"):

- Trecho "ad":

$$Mo_ad(x) := RVa \cdot x - \frac{q \cdot x^2}{2} = \frac{440}{3} \cdot x - \frac{20 \cdot x^2}{2}$$

$$Vo_ad(x) := RVa - q \cdot x = \frac{440}{3} - 20 \cdot x$$



* Considerar a origem de "x" ainda no ponto "a" para o trecho "db":

$$Mo_db(x) := RVa \cdot x - q \cdot Lad \cdot \left(x - \frac{Lad}{2}\right) - P \cdot (x - Lad) = \frac{440}{3} \cdot x - 20 \cdot 9 \cdot \left(x - \frac{9}{2}\right) - 50 \cdot (x - 9)$$

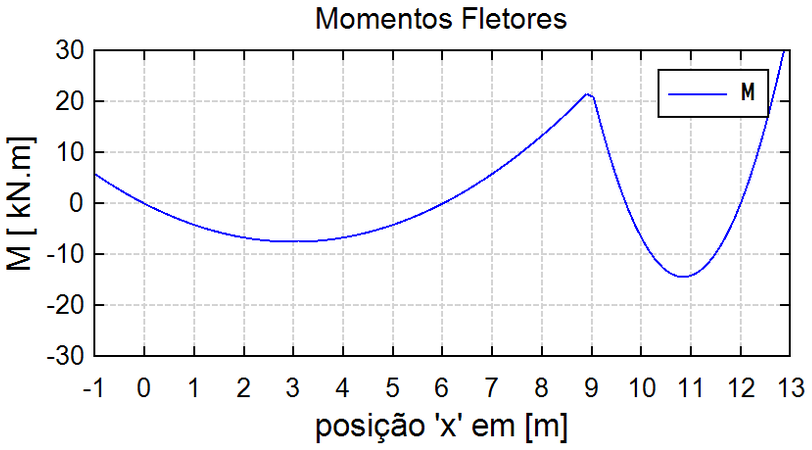
$$Vo_db(x) := RVa - q \cdot Lad - P = -83,3333$$

Momentos Fletores:

$$Mad(x) := Mo_ad(x) - H \cdot y(x) = \frac{440}{3} \cdot x - \frac{20 \cdot x^2}{2} - \frac{260}{3} \cdot \left(0 + \frac{7}{4} \cdot x + \left(-\frac{1}{8}\right) \cdot x^2\right)$$

$$Mdb(x) := Mo_db(x) - H \cdot y(x) = \frac{440}{3} \cdot x - 20 \cdot 9 \cdot \left(x - \frac{9}{2}\right) - 50 \cdot (x - 9) - \frac{260}{3} \cdot \left(0 + \frac{7}{4} \cdot x + \left(-\frac{1}{8}\right) \cdot x^2\right)$$

$M(x) := \text{if } x < Lad$
$\quad Mad(x)$
$\quad \text{else}$
$\quad Mdb(x)$



Esforços Cortantes e Axiais:

$$V_s(x) = V_{so}(x).cos(\alpha(x)) - H.sen(\alpha(x))$$
$$N_s(x) = -V_{so}(x).sen(\alpha(x)) - H.cos(\alpha(x))$$

$$\alpha(x) = arctg\left(\frac{dy(x)}{dx}\right)$$

$$\alpha(x) := \operatorname{atan}\left(\frac{d}{dx} y(x)\right) = -\operatorname{atan}\left(\frac{-7+x}{4}\right)$$

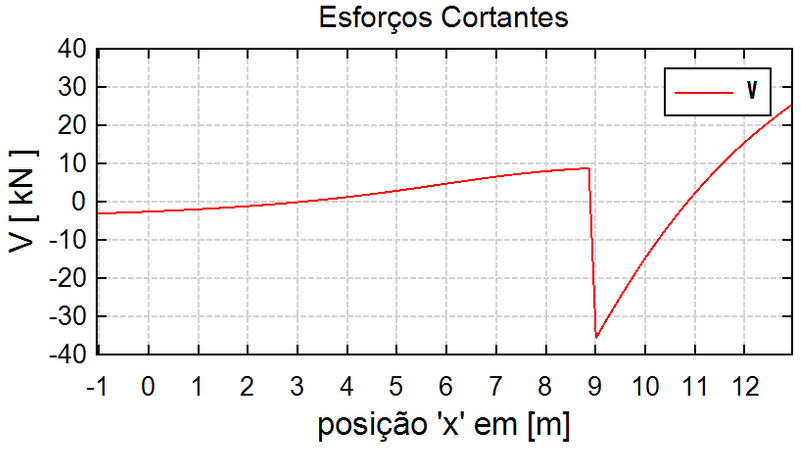
$$Vad(x) := Vo_ad(x)\cdot\cos(\alpha(x)) - H\cdot\sin(\alpha(x))$$

$$Vad(x) = \left(\frac{440}{3} - 20\cdot x\right)\cdot\cos\left(\operatorname{atan}\left(-\frac{128\cdot(-14+x+x)}{1024}\right)\right) - \frac{260}{3}\cdot\sin\left(\operatorname{atan}\left(-\frac{128\cdot(-14+x+x)}{1024}\right)\right)$$

$$Vdb(x) := Vo_db(x)\cdot\cos(\alpha(x)) - H\cdot\sin(\alpha(x))$$

$$Vdb(x) = \left(-\frac{250}{3}\right)\cdot\cos\left(\operatorname{atan}\left(-\frac{128\cdot(-14+x+x)}{1024}\right)\right) - \frac{260}{3}\cdot\sin\left(\operatorname{atan}\left(-\frac{128\cdot(-14+x+x)}{1024}\right)\right)$$

```
V(x) := if  x < Lad
          Vad(x)
        else
          Vdb(x)
```



$$V_s(x) = V_{so}(x).cos(\alpha(x)) - H.sen(\alpha(x))$$
$$N_s(x) = -V_{so}(x).sen(\alpha(x)) - H.cos(\alpha(x))$$

$Nad(x) := -Vo_ad(x) \cdot \sin(\alpha(x)) - H \cdot \cos(\alpha(x))$

$$Nad(x) = -\left(\frac{440}{3} - 20 \cdot x\right) \cdot \sin\left(\operatorname{atan}\left(-\frac{128 \cdot (-14 + x + x)}{1024}\right)\right) - \frac{260}{3} \cdot \cos\left(\operatorname{atan}\left(-\frac{128 \cdot (-14 + x + x)}{1024}\right)\right)$$

$Ndb(x) := -Vo_db(x) \cdot \sin(\alpha(x)) - H \cdot \cos(\alpha(x))$

$$Ndb(x) = -\left(-\frac{250}{3}\right) \cdot \sin\left(\operatorname{atan}\left(-\frac{128 \cdot (-14 + x + x)}{1024}\right)\right) - \frac{260}{3} \cdot \cos\left(\operatorname{atan}\left(-\frac{128 \cdot (-14 + x + x)}{1024}\right)\right)$$

$N(x) :=$ if $x < Lad$
$\quad Nad(x)$
else
$\quad Ndb(x)$

