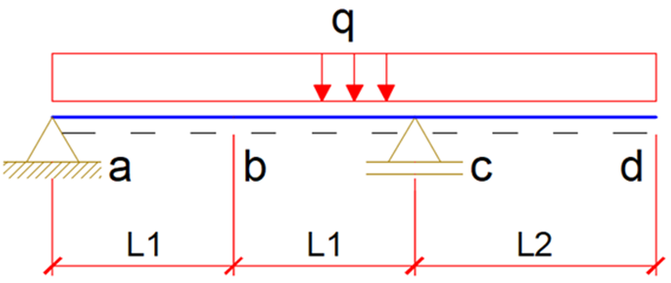
TE1_E09-01

- Calcular, considerando somente os efeitos de momentos fletores:

a) A deflexão do ponto "b";

b) A rotação do ponto "d";

c) A deflexão do ponto "d";



Dados:

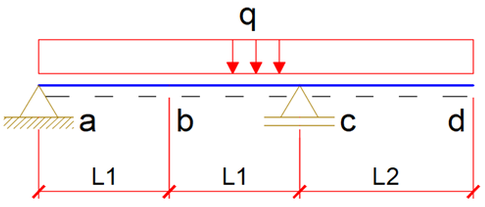
Todas as barras tem seção transversal retangular :

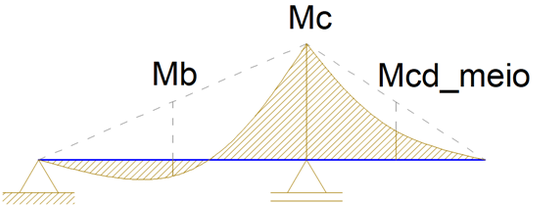
$b := 15\ cm$ (base) $h := 40\ cm$ (altura)

$E := 24000\ MPa$

$L1 := 3,0\ m$ $L2 := 4,0\ m$ $q := 50,0$ kN/m

Diagrama de Momentos Fletores do Carregamento Real:





- Sem calcular as reações de apoio:

$Ma := 0$ $Md := 0$ $Mc := -q \cdot \frac{L2^2}{2} = -400$

$Mb := \frac{Ma + Mc}{2} + \frac{q \cdot (2 \cdot L1)^2}{8} = 25$ $Mcd_meio := \frac{Mc + Md}{2} + \frac{q \cdot L2^2}{8} = -100$

- Calculando as reações de apoio:

ΣMc = 0 : (+) horário RVa.(L1+L1) - q.(L1+L1).L1 + q.L2.L2/2 = 0

$RVa := \frac{1}{L1 + L1} \cdot \left(q \cdot (L1 + L1) \cdot L1 - q \cdot L2 \cdot \frac{L2}{2} \right) = 83,333$

ΣFv = 0 : (+) para cima RVa + RVc - q.(L1+L1+L2) = 0

$RVc := q \cdot (L1 + L1 + L2) - RVa = 416,67$

$Mac(x) := RVa \cdot x - \frac{q \cdot x^2}{2} = \frac{250}{3} \cdot x - \frac{50 \cdot x^2}{2}$

$Ma := Mac(0) = 0$

$Mb := Mac(L1) = 25$

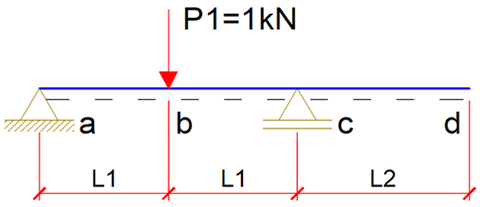
$Mc := Mac(2 \cdot L1) = -400$

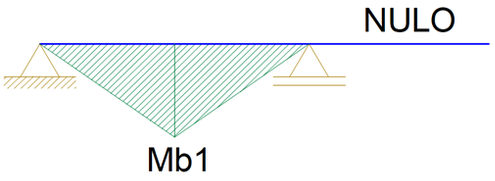
$Mcd(x) := RVa \cdot x - \frac{q \cdot x^2}{2} + RVc \cdot (x - 2 \cdot L1) = \frac{250}{3} \cdot x - \frac{50 \cdot x^2}{2} + \frac{1250}{3} \cdot (x - 2 \cdot 3)$

$Mc := Mcd(2 \cdot L1) = -400$

$Md := Mcd(2 \cdot L1 + L2) = 0$

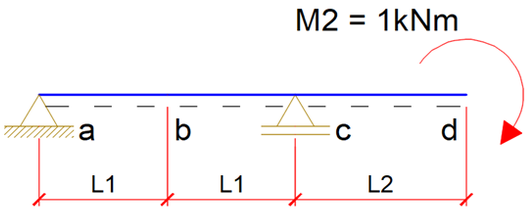
Diagrama de Momentos Fletores do Carregamento Virtual Relativo à Deflexão do Ponto "b":

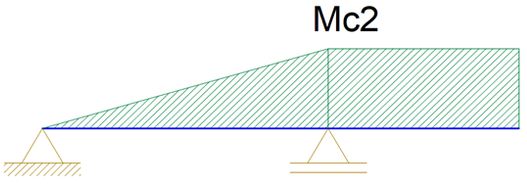




$P1 := 1$ $\quad Mb1 := \frac{P1 \cdot L1 \cdot L1}{L1 + L1} = 1,5$

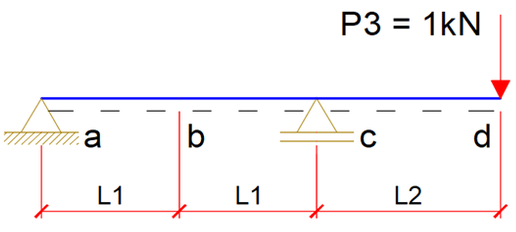
Diagrama de Momentos Fletores do Carregamento Virtual Relativo à Rotação do Ponto "d":

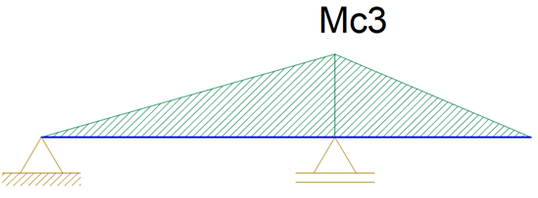




$M2 := 1$ $\quad Mc2 := -M2 = -1$

Diagrama de Momentos Fletores do Carregamento Virtual Relativo à Deflexão do Ponto "d":

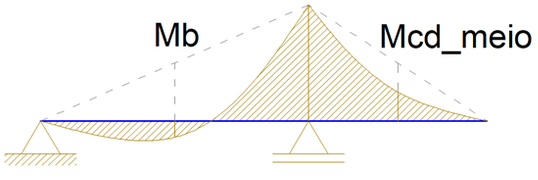




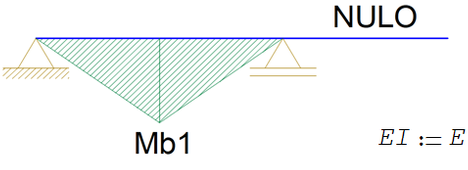
$P3 := 1$ $\quad Mc3 := -P3 \cdot L2 = -4$

Cálculo da Deflexão do Ponto "b":

M_real



M_virtual_δb

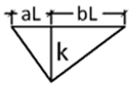


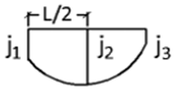
$$\bar{P}.\delta = \int_0^L \bar{M}\frac{M}{EI}dx$$

$EI := E \cdot \frac{b \cdot h^3}{12} = 19200\ \text{kN m}^2$

* Os efeitos de flexão do trecho "cd" nãon inerferem na deflexão do ponto "b";

Barra "ac":





1/6 Lk[j_1b+2j_2+j_3a -ab(j_1-2j_2+j_3)]

$j1 := 0$ $\quad j2 := Mb = 25$ $\quad j3 := Mc = -400$

$k := Mb1 = 1,5$ $\quad L := 2 \cdot L1 = 6$

* a.L + b.L = L logo: a+b = 1

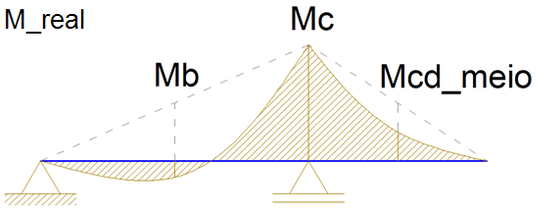
$a := 0,5$ $\quad b := 0,5$

$$\delta b := \frac{1}{6} \cdot L \cdot k \cdot \left(j1 \cdot b + 2 \cdot j2 + j3 \cdot a - a \cdot b \cdot \left(j1 - 2 \cdot j2 + j3\right)\right) \cdot \frac{1}{EI}\ \text{kN m}^2 = -0,0029297$$

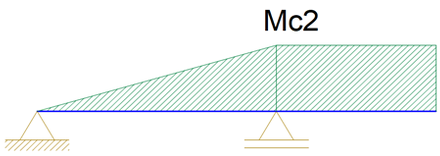
$\delta b := (-1000) \cdot \delta b = 2,9297$ mm para cima

Cálculo da Rotação do Ponto "d":

M_real

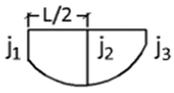


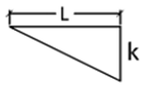
M_virtual_φd



$$\bar{P}.\delta = \int_0^L \bar{M}\frac{M}{EI}dx$$

Barra "ac":





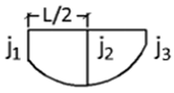
1/6 Lk(2j₂+j₃)

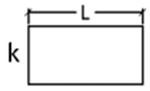
$j1 := 0$ $j2 := Mb = 25$ $j3 := Mc = -400$

$k := Mc2 = -1$ $L := 2 \cdot L1 = 6$

$\varphi d_ac := \frac{1}{6} \cdot L \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{EI}$ kN m^2 $= 0,0182$ rad horário

Barra "cd":





1/6 Lk(j₁+4j₂+j₃)

$j1 := Mc = -400$ $j2 := Mcd_meio = -100$ $j3 := 0$

$k := Mc2 = -1$ $L := L2 = 4$

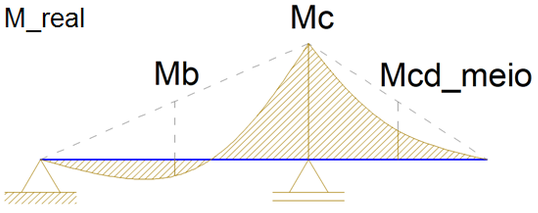
$\varphi d_cd := \frac{1}{6} \cdot L \cdot k \cdot (j1 + 4 \cdot j2 + j3) \cdot \frac{1}{EI}$ kN m^2 $= 0,0278$ rad horário

$\varphi d := \varphi d_ac + \varphi d_cd = 0,046007$ rad horário

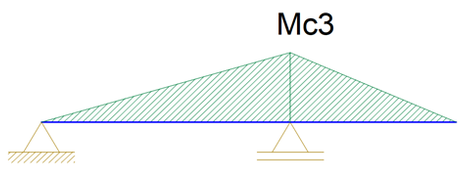
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Cálculo da Deflexão do Ponto "d":

M_real

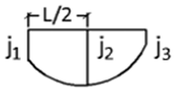


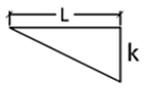
M_virtual_δd



$$\bar{P}.\delta = \int_0^L \bar{M}\frac{M}{EI}dx$$

Barra "ac":





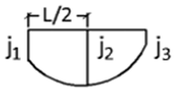
1/6 Lk(2j₂+j₃)

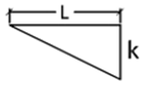
$j1 := 0$ $j2 := Mb = 25$ $j3 := Mc = -400$

$k := Mc3 = -4$ $L := 2 \cdot L1 = 6$

$\delta d_ac := \frac{1}{6} \cdot L \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{EI}$ kN m^2 $= 0,0729$ m descendo

Barra "cd":





1/6 Lk(2j₂+j₃)

$j1 := 0 = 0$ $j2 := Mcd_meio = -100$ $j3 := Mc = -400$

$k := Mc3 = -4$ $L := L2 = 4$

$\delta d_cd := \frac{1}{6} \cdot L \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{EI}$ kN m^2 $= 0,0833$ m descendo

$\delta d := 1000 \cdot (\delta d_ac + \delta d_cd) = 156,2$ mm descendo

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