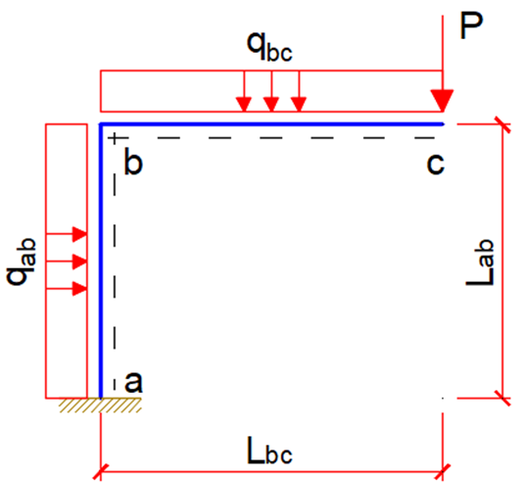
TE1_E09-03

- Calcular o deslocamento vertical do ponto "c".



$Lbc := 5,0$ m $qbc := 10,0$ kN/m

$Lab := 4,0$ m $qab := 5,0$ kN/m

$P := 10,0$ kN

- Barra "bc": seção transversal retangular:

$b_bc := 0,20$ m $h_bc := 0,40$ m

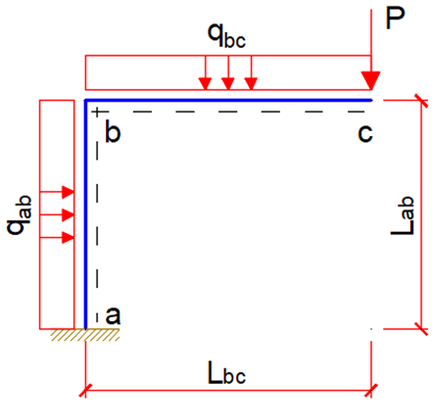
- Barra "ab": seção transversal retangular:

$b_ab := 0,35$ m $h_ab := 0,35$ m

- Módulo e elasticidade e coef. de Poisson para todas as barras:

$E := 40 \cdot 10^{6}$ kN/m² $\nu := 0,2$

Equações de Momento, Cortante e Axial para o Carregamento Real:



Como a estrutura é de fato uma viga em balanço com um trecho vertical, é possível escrever as equações de esforços internos sem o cálculo das reações de apoio.

- Equações para a barra "cb" (origem do eixo "x" no ponto "c"):

$$Mcb(x) := -P \cdot x - \frac{qbc \cdot x^{2}}{2} = -10 \cdot x - \frac{10 \cdot x^{2}}{2}$$

$$Vcb(x) := -\frac{d}{dx} Mcb(x) = 10 \cdot (1 + x)$$

$$Ncb(x) := 0$$

$$Mb := Mcb(Lbc) = -175$$

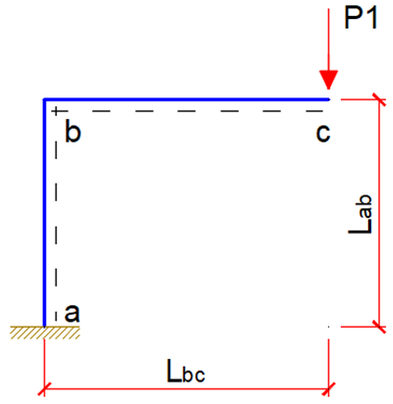
- Equações para a barra "ba" (origem do eixo "x" no ponto "b"):

$$Mba(x) := Mb - \frac{qab \cdot x^{2}}{2} = -175 - \frac{5 \cdot x^{2}}{2}$$

$$Vba(x) := -\frac{d}{dx} Mba(x) = 5 \cdot x$$

$$Nba(x) := -P - qbc \cdot Lbc = -60$$

Equações de Momento, Cortante e Axial para o Carregamento Virtual relativo ao deslocamento vertical do ponto "c":



- Equações para a barra "cb" (origem do eixo "x" no ponto "c"): $P1 := 1$

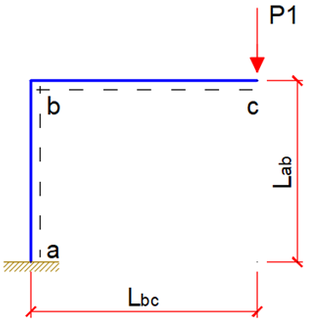
$$M1cb(x) := -P1 \cdot x = -1 \cdot x$$

$$V1cb(x) := -\frac{d}{dx} M1cb(x) = 1$$

$$N1cb(x) := 0$$

$$M1b := M1cb(Lbc) = -5$$

- Equações para a barra "ba" (origem do eixo "x" no ponto "b"):



$M1ba(x) := M1b = -5$

$V1ba(x) := -\frac{d}{dx} M1ba(x) = 0$

$N1ba(x) := -P1 = -1$

Cálculo do deslocamento vertical no ponto "c" utilizando o PTV via equações:

$G := \frac{E}{2\cdot(1+\nu)} = 1,6667\cdot 10^{7}$ $A_bc := b_bc \cdot h_bc = 0,08$ $A_ab := b_ab \cdot h_ab = 0,1225$

$\chi := \frac{6}{5}$ $I_bc := \frac{b_bc \cdot h_bc^{3}}{12} = 0,0010667$ $I_ab := \frac{b_ab \cdot h_ab^{3}}{12} = 0,0012505$

$$\overline{P}.\delta = \int_0^L \overline{N}\frac{N}{EA}dx + \int_0^L \overline{M}\frac{M}{EI}dx + \chi\int_0^L \overline{V}\frac{V}{GA}dx$$

Efeito de momento fletor:

$mab := \int_0^{Lab} \frac{Mba(x)\cdot M1ba(x)}{E\cdot I_ab}dx = 0,0753$ $mbc := \int_0^{Lbc} \frac{Mcb(x)\cdot M1cb(x)}{E\cdot I_bc}dx = 0,0281$

$\delta_vert_c_mom := mab + mbc = 0,10338$

$Vcb(x) = 10\cdot(1+x)$ $V1cb(x) = 1$

Efeito de esforço cortante:

$vab := \chi\cdot\int_0^{Lab} \frac{Vba(x)\cdot V1ba(x)}{G\cdot A_ab}dx = 0,0000$ $vbc := \chi\cdot\int_0^{Lbc} \frac{Vcb(x)\cdot V1cb(x)}{G\cdot A_bc}dx = 0,00015750$

$\delta_vert_c_cor := vab + vbc = 0,00015750$

Efeito de esforço axial:

$nab := \int_0^{Lab} \frac{Nba(x)\cdot N1ba(x)}{E\cdot A_ab}dx = 4,8980\cdot 10^{-5}$ $nbc := \int_0^{Lbc} \frac{Ncb(x)\cdot N1cb(x)}{E\cdot A_bc}dx = 0,0000$

$\delta_vert_c_axi := nab + nbc = 4,8980\cdot 10^{-5}$

$\delta_vert_c := \delta_vert_c_mom + \delta_vert_c_cor + \delta_vert_c_axi = 0,10358$

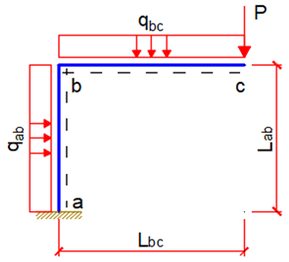
Erro ao considerar somente efeitos de flexão:

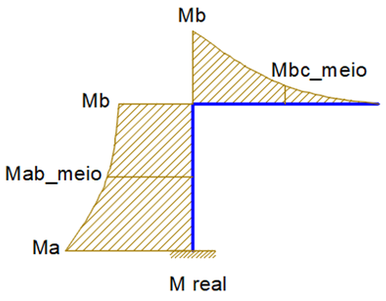
$erro_relat := \frac{\delta_vert_c}{\delta_vert_c_mom} = 1,002$

Cálculo do deslocamento vertical no ponto "c" utilizando o PTV via integração por tabelas:

$$\overline{P}.\delta = \int_0^L \overline{N}\frac{N}{EA}dx + \int_0^L \overline{M}\frac{M}{EI}dx + \chi\int_0^L \overline{V}\frac{V}{GA}dx$$

Efeito de momento fletor:





M real

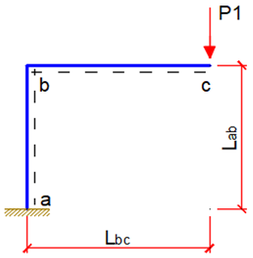
$$Mb := -P\cdot Lbc - \frac{qbc\cdot Lbc^{2}}{2} = -175$$

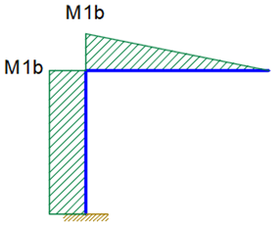
$$Ma := Mb - qab\cdot\frac{Lab^{2}}{2} = -215$$

$$Mbc_meio := -P\cdot\frac{Lbc}{2} - \frac{qbc\cdot\left(\frac{Lbc}{2}\right)^{2}}{2} = -56{,}25$$

$$Mab_meio := \frac{Ma+Mb}{2} + \frac{qab\cdot Lab^{2}}{8} = -185$$

$$Mab_meio := Mb - \frac{qab\cdot\left(\frac{Lab}{2}\right)^{2}}{2} = -185$$

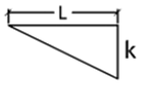


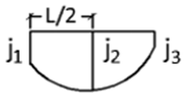


M virtual
(desloc. vertical em "c")

$$M1b := -P1\cdot Lbc = -5$$

Barra "bc":





1/6 Lk(2j₂+j₃)

Como "k" está para direita na tabela é necessário inverter a posição dos "j"s

$$k := M1b = -5 \qquad j2 := Mbc_meio = -56{,}25$$

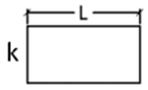
* j3 é o valor no diagrama de momentos fletores parabólico que está para o mesmo lado de "k" no diagrama triangular.

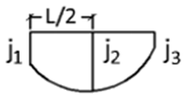
$$j3 := Mb \qquad j1 := 0$$

$$mbc = 0{,}028076$$

$$mbc := \frac{1}{6}\cdot Lbc\cdot k\cdot\left(2\cdot j2 + j3\right)\cdot\frac{1}{E\cdot I_bc} = 0{,}028076$$

Barra "ab":





1/6 Lk(j₁+4j₂+j₃)

Como o diagrama virtual é retangular não há problema em inverter os "j"s

$$k := M1b = -5$$

$$j1 := Ma = -215 \qquad j2 := Mab_meio = -185 \qquad j3 := Mb = -175$$

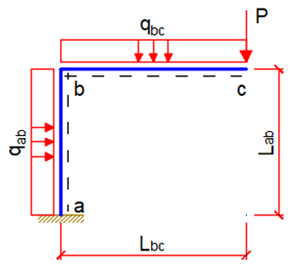
$$mab = 0{,}0753$$

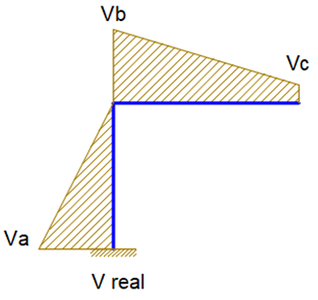
$$mab := \frac{1}{6}\cdot Lab\cdot k\cdot\left(j1 + 4\cdot j2 + j3\right)\cdot\frac{1}{E\cdot I_ab} = 0{,}0753$$

$$\delta_vert_c_mom = 0{,}10338$$

$$\delta_vert_c_mom_tab := mab + mbc = 0{,}10338$$

Efeito de esforço cortante:

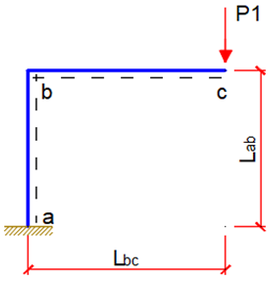


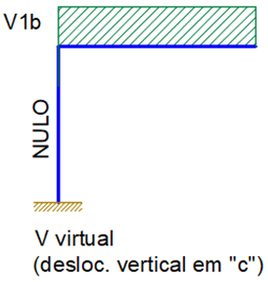


$Vc := P = 10$

$Vb := P + qbc \cdot Lbc = 60$

$Va := qab \cdot Lab = 20$



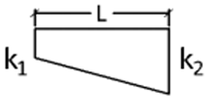


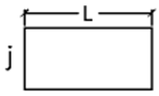
$V1b := P1 = 1$

Barra "ab": $vab = 0$

$vab := 0$

Barra "bc":





1/2 Lj(k₁+k₂)

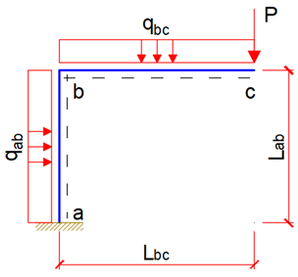
$k1 := Vb = 60$ $k2 := Vc = 10$

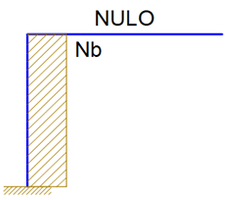
$j := V1b = 1$

$vbc = 0,0001575$

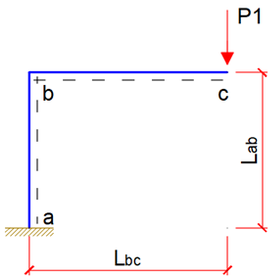
$vbc := \chi \cdot \frac{1}{2} \cdot Lbc \cdot j \cdot (k1 + k2) \cdot \frac{1}{G \cdot A_bc} = 0,0001575$

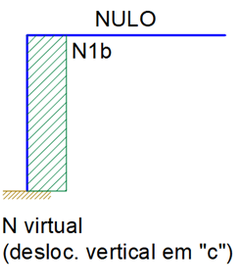
Efeito de esforço axial:





$Nb := -P - qbc \cdot Lbc = -60$



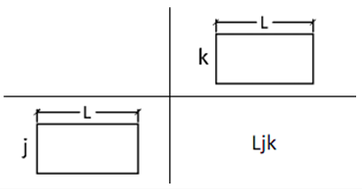


$N1b := -P1 = -1$

Barra "bc": $nbc = 0$

$nbc := 0$

Barra "ab":



$j := N1b = -1$ $k := Nb = -60$

$nab = 4,898 \cdot 10^{-5}$

$nab := Lab \cdot j \cdot k \cdot \frac{1}{E \cdot A_ab} = 4,8980 \cdot 10^{-5}$