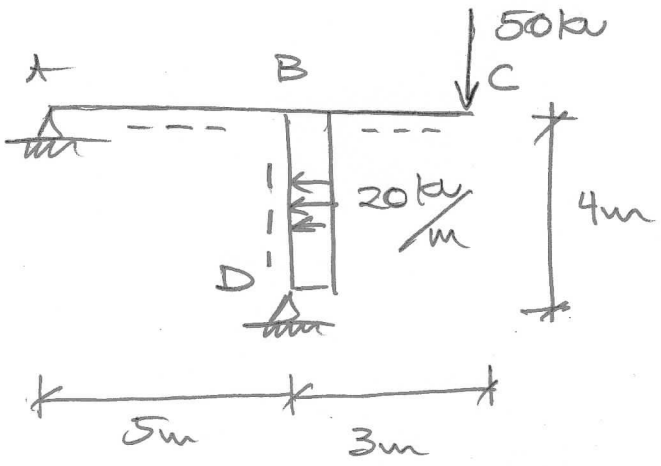
TE2-4.5

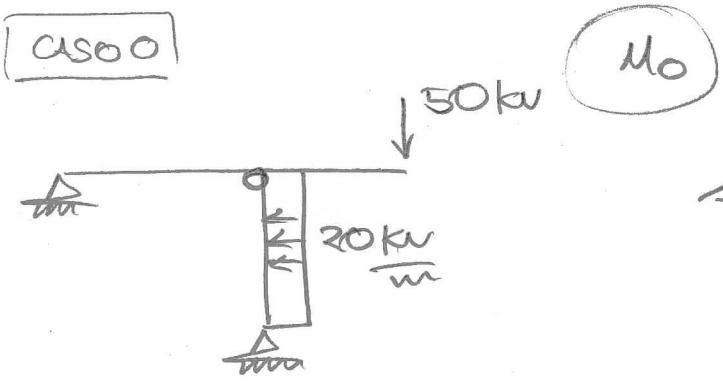
- APLICANDO O MÉTODO DAS FORÇAS CALCULE AS REAÇÕES DE APOIO DO MODELO ESTRUTURAL DE GRAU ESTÁTICO 1 ABAIXO;



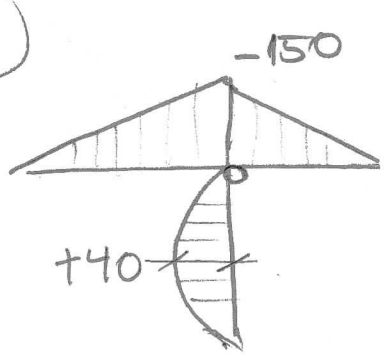
* CONSIDERAR SOMENTE EFEITOS DE FLEXÃO
* TODAS AS BARRAS POSSUEM O MESMO MÓDULO DE RIGIDEZ FLEXURAL EI;

SOLUÇÃO 1:

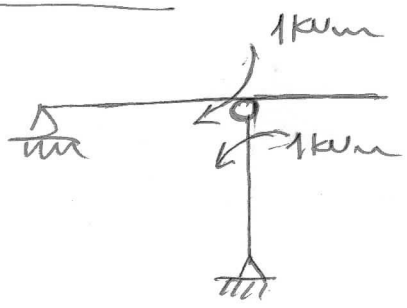
CASO 0



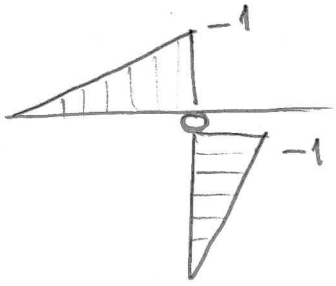
M_0



CASO 1



M_1



$$\phi_{10} = \frac{1}{EI}\left[\frac{1}{3}\times 5\times -150\times -1 + \frac{1}{6}\times 4\times -1(2\times 40+0)\right]$$

$$= \frac{1}{EI}\left(+250 - 160/3\right) = \frac{590}{3EI}$$

$$f_{11} = \frac{1}{EI}\left[\frac{1}{3}\times 5\times -1\times -1 + \frac{1}{3}\times 4\times -1\times -1\right]$$

$$= \frac{1}{EI}\left[\frac{5}{3} + \frac{4}{3}\right] = \frac{3}{EI}$$

$$\varphi_B = \varphi_{10} + f_{11}\times M_B \Rightarrow M_B = \frac{\varphi_B - \varphi_{10}}{f_{11}}$$

$$M_B = \frac{0 - 590/3EI}{3/EI} = \frac{590}{9}\,kNm = \underline{65{,}556\,kNm}$$

$$\Sigma M_B^{BD} = 0 \circlearrowright (+) \quad 65{,}556\,kNm + 20\,\frac{kN}{m}\times\frac{(4m)^2}{2} - H_D\times 4m = 0$$

$$H_D = \frac{1}{4}\times\left(590/9 + 10\times 16\right) = \frac{1015}{18}\,kN = \underline{56{,}389\,kN} \rightarrow$$

$$\Sigma M_A = 0 \circlearrowleft (+) \quad V_D\times 5m - 50\,kN\times 8m - 20\,\frac{kN}{m}\times\frac{(4m)^2}{2} + \frac{1015}{18}\,kN\times 4m = 0$$

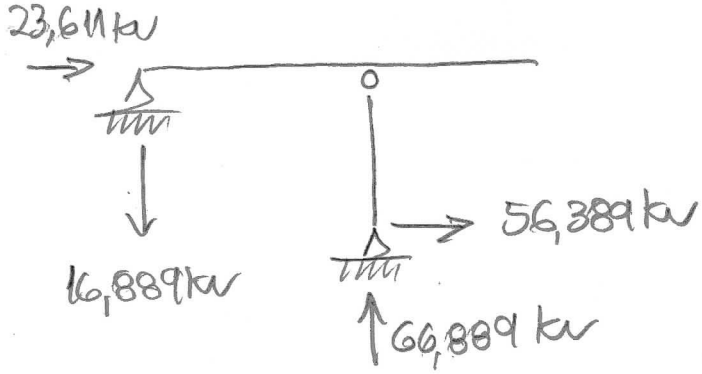
$$V_D = \frac{1}{5}\left(+50\times 8 + 20\times 8 - 1015/18\times 4\right) = \frac{602}{9}\,kN = \underline{66{,}889\,kN} \uparrow$$

$$\Sigma F_H = 0 \rightarrow (+) \quad H_A - 20\,\frac{kN}{m}\times 4m + \frac{1015}{18}\,kN = 0$$

$$H_A = 80 - 1015/18 = \frac{425}{18}\,kN = 23{,}611\,kN \rightarrow$$

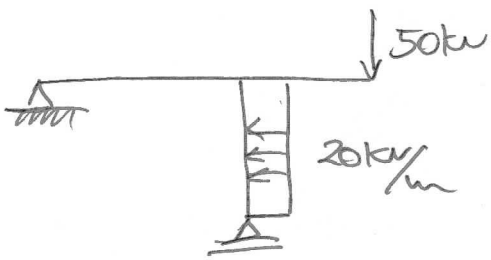
$$\Sigma F_V = 0 \uparrow (+) \quad V_A + 602/9\,kN - 50\,kN = 0$$

$$V_A = 50 - 602/9 = -152/9\,kN = \underline{-16{,}889\,kN} \downarrow$$

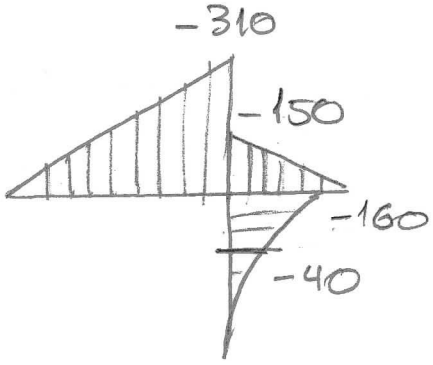


Solução 2:

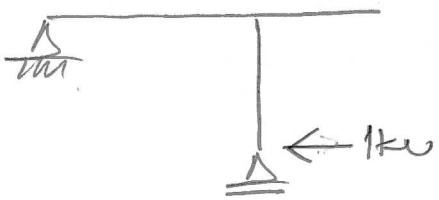
CASO 0



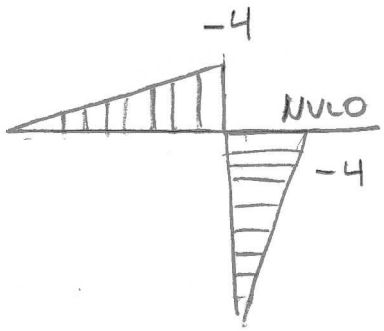
M_0



CASO 1



M_1



$$\delta_{10} = \frac{1}{EI}\left[\frac{1}{3}5\times-310\times-4 + \frac{1}{6}\times4\times-4(2\times-40-160)\right]$$

$$= \frac{1}{EI}\left[\frac{6200}{3} + 640\right] = \frac{8120}{3EI}$$

$$f_{11} = \frac{1}{EI}\left[\frac{1}{3}5\times-4\times-4 + \frac{1}{3}4\times-4\times-4\right] =$$

$$= \frac{1}{EI}\left[\frac{80}{3} + \frac{64}{3}\right] = \frac{48}{EI}$$

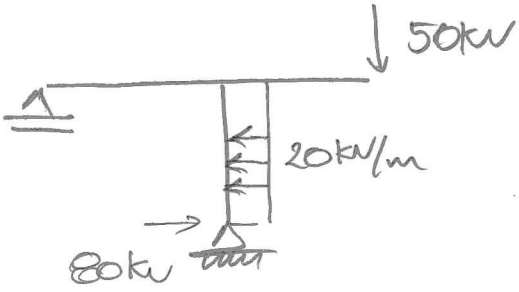
$$\delta_D = \delta_{10} + f_{11}\times H_D \qquad H_D = -\delta_{10}/f_{11}$$

$$H_D = \frac{-8120/3EI}{48/EI} = \frac{1015}{18} = 56{,}389\ kN \rightarrow$$

IGUAL À SOLUÇÃO 1

Solução 3:

CASO 0



$H_D = 20\,kN/m \times 4m = 80\,kN$

$\Sigma M_D = 0 \quad \circlearrowleft +$

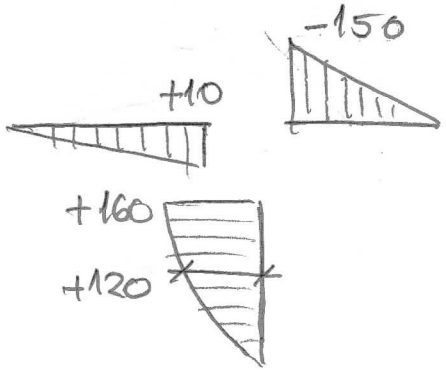
$V_A \times 5m + 50kN \times 3m - 20\frac{kN}{m} \times \frac{(4m)^2}{2} = 0$

$V_A = \frac{1}{5}(-50 \times 3 + 10 \times 16) = 2\,kN \uparrow$

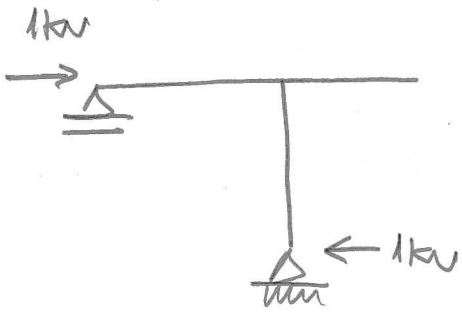
$M_B^{BD} = 80 \times 4 - 20 \times \frac{4^2}{2} = +160$

$M_{BD} = 80 \times 2 - 20 \times \frac{2^2}{2} = 120$

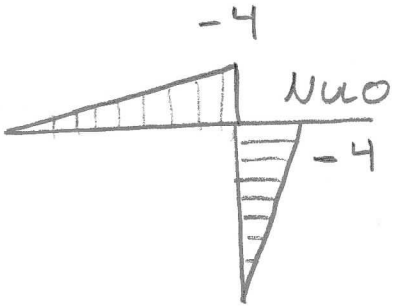
M_0



CASO 1



M_1



$\delta_{10} = \frac{1}{EI}\left[\frac{1}{3} 5 \times 10 \times -4 + \frac{1}{6} \times 4 \times -4 (2 \times 120 + 160)\right]$

$= \frac{1}{EI}\left[-200/3 - 3200/3\right] = -\frac{3400}{3EI}$

$f_{11} = \frac{1}{EI}\left[\frac{1}{3} \times 5 \times -4 \times -4 + \frac{1}{3} \times 4 \times -4 \times -4\right]$

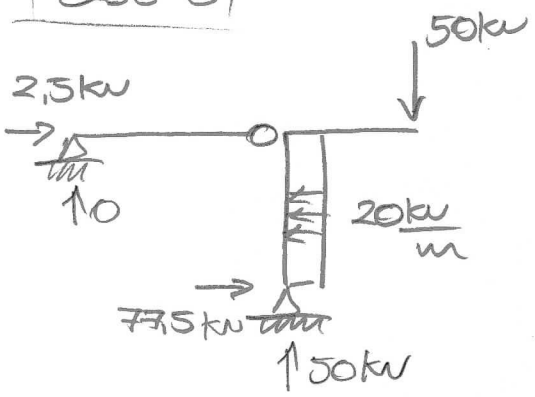
$= \frac{1}{EI}\left[80/3 + 64/3\right] = 48/EI$

$H_A = -\delta_{10}/f_{11} = \frac{3400/3EI}{48/EI} = \frac{425}{18} = 23{,}611\,kN \rightarrow$

IGUAL À SOLUÇÃO 1;

Solução 4:

CASO 0



$\sum M_B^{\rightarrow D} = 0 \circlearrowleft (+)$

$H_D \times 4m - 50kN \times 3m - 20\frac{kN}{m} \times \frac{(4m)^2}{2} = 0$

$H_D = \frac{1}{4}(50 \times 3 + 10 \times 16) = \frac{155}{2}\ kN$

$H_D = 77{,}5\ kN \rightarrow$

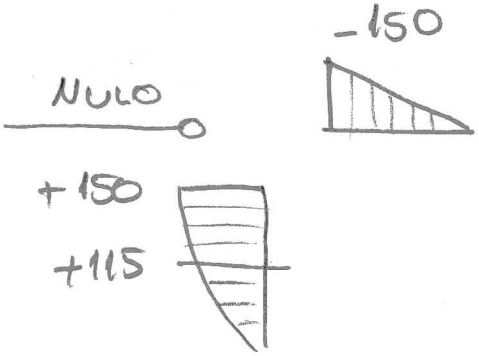
$M_B^{BD} = 77{,}5 \times 4 - 20 \times \frac{4^2}{2}$

$M_B^{BD} = 150\ kNm$

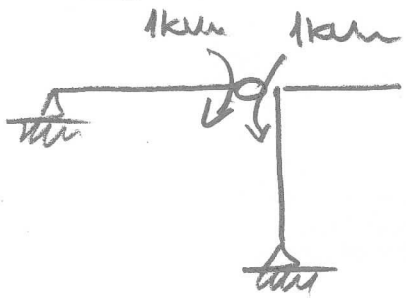
$M_{BD} = 77{,}5 \times 2 - 20 \times \frac{2^2}{2}$

$M_{BD} = 115\ kNm$

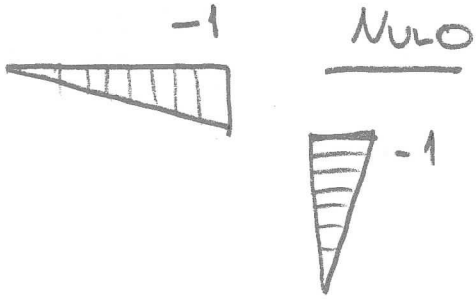
M_0



CASO 1



M_1



$\delta_{10} = \frac{1}{EI}\left[\frac{1}{6} \times 4 \times -1\ (2 \times 115 + 150)\right] = \frac{-760}{3EI}$

$f_{11} = \frac{1}{EI}\left[\frac{1}{3} \times 4 \times -1 \times -1 + \frac{1}{3} \times 5 \times -1 \times -1\right] = \frac{3}{EI}$

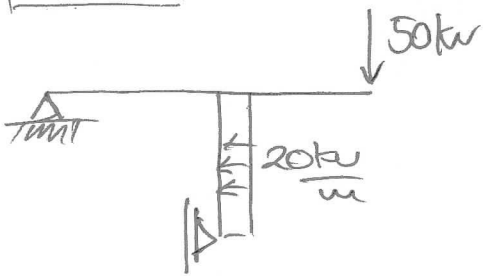
$M_B^{AB} = -\frac{\delta_{10}}{f_{11}} = \frac{760/3EI}{3/EI} = \frac{760}{9}\ kNm = 84{,}444\ kNm$

$\sum M_B^{AB} = 0 \circlearrowleft (+) \quad V_A \times 5m + 760/9\ kNm = 0$

$V_A = -\frac{760}{9} \times \frac{1}{5} = -\frac{152}{9}\ kN = -16{,}889\ kN$ IGUAL À SOLUÇÃO 1

Solução 5:

CASO 0



$\Sigma M_A = 0$ ↺ +

$H_D \times 4m - 50kN \times 8m - 20\frac{kN}{m} \times (4m)^2/2 = 0$

$H_D = 1/4\,(50 \times 8 + 20 \times 8) = 140kN \rightarrow$

$M_B^{AB} = -50kN \times 3m - 20\frac{kN}{m} \times (4m)^2/2$

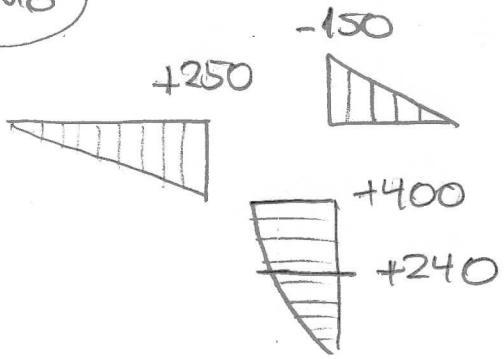
$+140kN \times 4m = +250kNm$

$M_B^{BD} = -20\frac{kN}{m} \times \frac{(4m)^2}{2} + 140kN \times 4m$

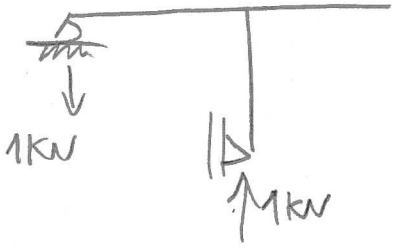
$M_B^{BD} = +400kNm$

$M_{BD} = 140 \times 2 - 20 \times 2^2/2 = 240kNm$

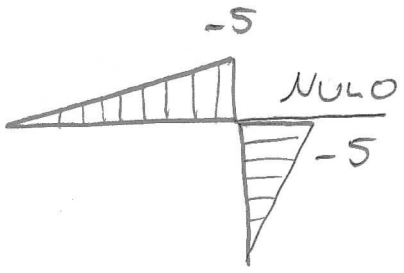
M_0



CASO 1



M_1



$\delta_{10} = 1/EI\left[1/3(5)\times(-5)\times(+250) + 1/6\,4\times(-5)\times(2\times240+400)\right]$

$= 1/EI\left[-\frac{6250}{3} - \frac{8800}{3}\right] = -\frac{15050}{3EI}$

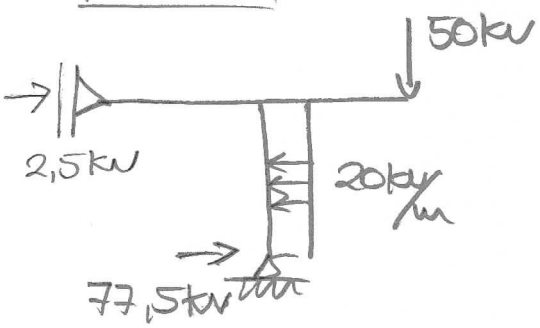
$f_{11} = 1/EI\left[1/3\,5\times-5\times-5 + 1/3\,4\times-5\times-5\right] = 75/EI$

$V_D = \frac{-\delta_{10}}{f_{11}} = \frac{+15050/3EI}{75/EI} = \frac{602}{9} = 66,889kN \uparrow$

IGUAL À SOLUÇÃO 1

Solução 6:

Caso 0



$\Sigma M_D = 0 \circlearrowleft +$

$H_A \times 4m + 20kN/m \times \frac{(4m)^2}{2} - 50kN \times 3m = 0$

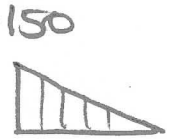
$H_A = \frac{1}{4}(-10 \times 16 + 50 \times 3) = -\frac{5}{2}\ kN$

$\Sigma F_H = 0 \rightarrow +$

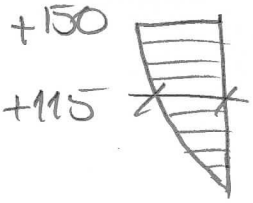
$H_D - 20kN/m \times 4m + 2,5kN = 0$

$H_D = +20 \times 4 - 2,5 = 77,5\ kN \rightarrow$

M_0



Nulo

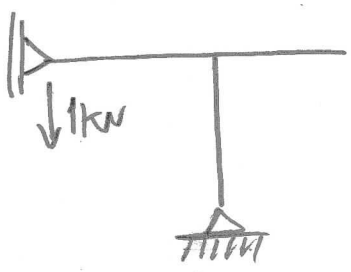


$M_B^{BD} = -20\frac{kN}{m} \times \frac{(4m)^2}{2} + 77,5kN \times 4m$

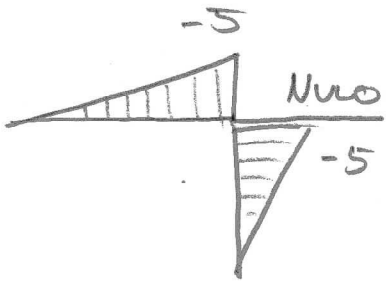
$M_B^{BD} = +150\ kNm$

$M_{BD} = 77,5 \times 2 - 20 \times \frac{2^2}{2} = 115$

Caso 1



M_1



$\delta_{10} = \frac{1}{EI}\left[\frac{1}{6} \times 4 \times (-5) \times (2 \times 115 + 150)\right] = -\frac{3800}{3EI}$

$f_{11} = \frac{1}{EI}\left[\frac{1}{3}5 \times -5 \times -5 + \frac{1}{3}4 \times -5 \times -5\right] = \frac{75}{EI}$

$V_A = \frac{-\delta_{10}}{f_{11}} = \frac{3800/3EI}{75/EI} = \frac{152}{9} = 16,889kN \downarrow$

Igual à solução 1