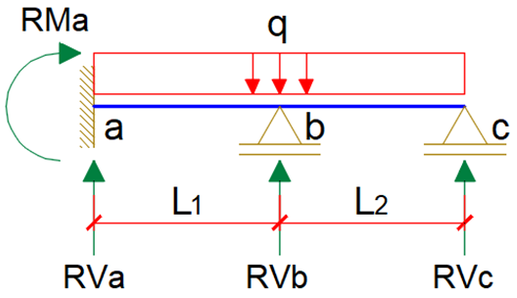
TE2_E05-01

- Determinar as reações de apoio da estrutura hiperestática ao lado (considerando somente efeitos de flexão):



- Dados:

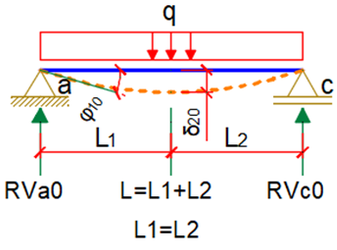
$E := 20000\ \text{MPa}$ $\quad L1 := 4,0\ \text{m}$

$b1 := 20\ \text{cm}$ $\quad L2 := 4,0\ \text{m}$

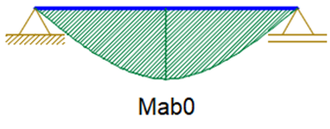
$h1 := 60\ \text{cm}$ $\quad q := 10\ \frac{\text{kN}}{\text{m}}$

Montagem do Sistema Principal e dos Casos Auxiliares - Diagramas de Momentos Fletores dos Casos:

Caso 0 (Sistema Principal)



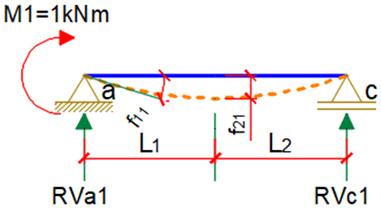
M0



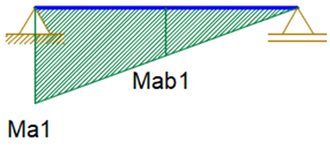
$L := L1 + L2 = 8\ \text{m}$

$Mab0 := \frac{q \cdot L^{2}}{8} = 80\ \text{kN m}$

Caso 1



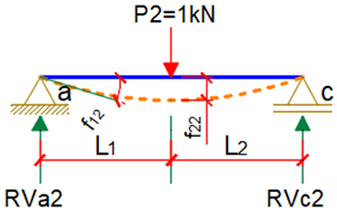
M1



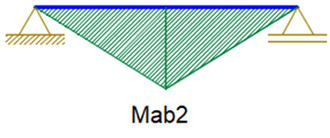
$Ma1 := 1\ \text{kN m}$

$Mab1 := \frac{Ma1}{2} = 0,5\ \text{kN m}$

Caso 2



M2

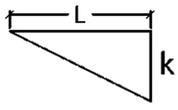


$P2 := 1\ \text{kN}$

$Mab2 := \frac{P2 \cdot L1 \cdot L2}{L1 + L2} = 2\ \text{kN m}$

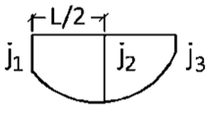
Combinando os diagramas M0 e M1:

$\varphi 10 := \frac{1}{EI} \cdot \int_{0}^{L} M0(x) \cdot M1(x)\,dx$ $\quad EI := \frac{E \cdot b1 \cdot h1^{3}}{12} = 72000\ \text{kN m}^{2}$



$k := Ma1 = 1\ \text{kN m}$ $\quad j2 := Mab0 = 80\ \text{kN m}$

$j1 := 0\ \text{kN m}$ $\quad j3 := 0\ \text{kN m}$

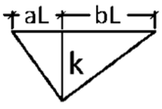


1/6 Lk(2j₂+j₃)

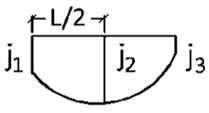
$\varphi 10 := \frac{1}{EI} \cdot \frac{1}{6} \cdot L \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{1\ \text{kN}} = 0,002963\ \text{m}$

Combinando os diagramas M0 e M2:

$\delta 20 := \frac{1}{EI} \cdot \int_{0}^{L} M0(x) \cdot M2(x)\,dx$



$a := 0,5$ $\quad b := 0,5$ $\quad L = 8\ \text{m}$ $\quad k := Mab2 = 2\ \text{kN m}$

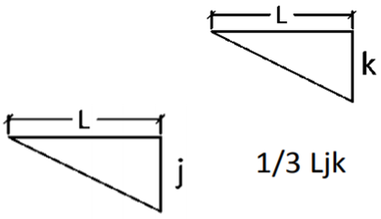


1/6 Lk[j₁b+2j₂+j₃a -ab(j₁-2j₂+j₃)]

$\delta 20 := \frac{1}{EI} \cdot \frac{1}{6} \cdot L \cdot k \cdot (j1 \cdot b + 2 \cdot j2 + j3 \cdot a - a \cdot b \cdot (j1 - 2 \cdot j2 + j3)) \cdot \frac{1}{\text{kN}}$

$\delta 20 = 0,0074074\ \text{m}$

Combinando os diagramas M1 e M1:



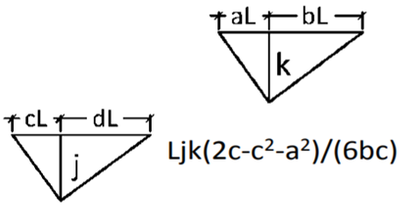
1/3 Ljk

$$f11 := \frac{1}{EI} \cdot \int_0^L M1(x) \cdot M1(x)\,dx$$

$$k := Ma1 = 1\ kN\ m \qquad j := Ma1 = 1\ kN\ m$$

$$f11 := \frac{1}{EI} \cdot \frac{1}{3} \cdot L \cdot j \cdot k \cdot \frac{1}{1\ kN} = 3{,}7037 \cdot 10^{-5}\ m$$

Combinando os diagramas M2 e M2:



Ljk(2c-c²-a²)/(6bc)

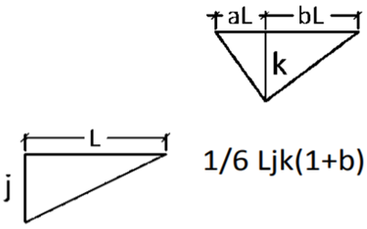
$$f22 := \frac{1}{EI} \cdot \int_0^L M2(x) \cdot M2(x)\,dx$$

$$c := a = 0{,}5 \qquad d := b = 0{,}5$$

$$k := Mab2 = 2\ kN\ m \qquad j := Mab2 = 2\ kN\ m$$

$$f22 := \frac{1}{EI} \cdot L \cdot j \cdot k \cdot \frac{\left(2 \cdot c - c^2 - a^2\right)}{6 \cdot b \cdot c} \cdot \frac{1}{1\ kN} = 0{,}0001\ m$$

Combinando os diagramas M1 e M2:



1/6 Ljk(1+b)

$$f12 := \int_0^L M1(x) \cdot M2(x)\,dx$$

$$a = 0{,}5 \qquad b = 0{,}5 \qquad k := Mab2 = 2\ kN\ m \qquad j := Ma1 = 1\ kN\ m$$

$$f12 := \frac{1}{EI} \cdot \frac{1}{6} \cdot L \cdot j \cdot k \cdot (1 + b) \cdot \frac{1}{1\ kN} = 5{,}5556 \cdot 10^{-5}\ m$$

Composição do Sistema de Equações de Compatibilidade e Cálculo das Redundantes Hiperestáticas Escolhidas:

$$\varphi 10 + R1 \cdot f11 + R2 \cdot f12 = 0 \qquad \text{--->} \qquad R1 \cdot f11 + R2 \cdot f12 = -\varphi 10$$

$$\delta 20 + R1 \cdot f21 + R2 \cdot f22 = 0 \qquad R1 \cdot f21 + R2 \cdot f22 = -\delta 20$$

Resolvendo o sistema de equações por substituição:

Isolando R1 na primeira equação:

$$R1 = \frac{-\varphi 10 - R2 \cdot f12}{f11}$$

Substituíndo R1 na segunda equação:

$$\frac{-\varphi 10 - R2 \cdot f12}{f11} \cdot f21 + R2 \cdot f22 = -\delta 20$$

Isolando e calculando o valor de R2:

$$\frac{-\varphi 10}{f11} \cdot f21 - \frac{R2 \cdot f12 \cdot f21}{f11} + R2 \cdot f22 = -\delta 20$$

$$R2 \cdot \left(-\frac{f12^2}{f11} + f22\right) = -\delta 20 + \frac{\varphi 10}{f11} \cdot f21$$

$$R2 := \frac{-\delta 20 + \frac{\varphi 10}{f11} \cdot f12}{-\frac{f12^2}{f11} + f22} = -45{,}714$$

"RVb é orientado para cima"

Substituíndo R2 calculado na expressão de R1:

$$R1 := \frac{-\varphi 10 - R2 \cdot f12}{f11} = -11{,}4286$$

"RMa gira antihorário"

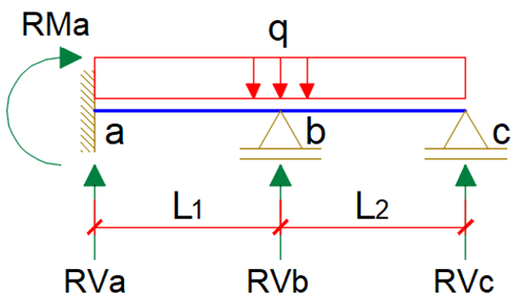
Resolvendo o sistema de equações por meio eletrônico:

$R1 \cdot f11 + R2 \cdot f12 = -\varphi 10$

$R1 \cdot f21 + R2 \cdot f22 = -\delta 20$

$$A := \begin{bmatrix} f11 & f12 \\ f12 & f22 \end{bmatrix} \qquad b := \begin{bmatrix} -\varphi 10 \\ -\delta 20 \end{bmatrix} \qquad R := A^{-1} \cdot b = \begin{bmatrix} -11,429 \\ -45,714 \end{bmatrix}$$

Cálculo das Demais Reações de Apoio:



$RVb := -R2 \cdot 1\ \text{kN} = 45,7143\ \text{kN}$

$RMa := R1 \cdot 1\ \text{kN m} = -11,4286\ \text{kN m}$

Cálculo de RVa: $\Sigma Mc = 0$ (+) horário

$$RVa \cdot L + RMa + RVb \cdot L2 - \frac{q \cdot L^2}{2} = 0$$

$$RVa := \frac{1}{L} \cdot \left(-RMa - RVb \cdot L2 + \frac{q \cdot L^2}{2} \right) = 18,571\ \text{kN}$$

Cálculo de RVc: $\Sigma Fv = 0$ (+) para cima

$RVa + RVb + RVc - q \cdot L = 0$

$RVc := -RVa - RVb + q \cdot L = 15,714\ \text{kN}$
