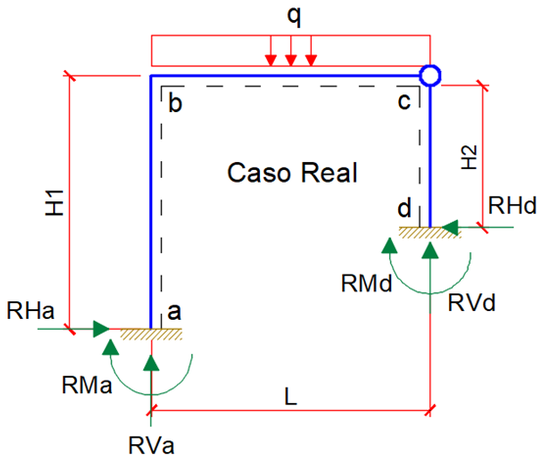
TE2_E05-02a

- Determinar as reações de apoio da estrutura hiperestática ao lado (considerando somente efeitos de flexão):



- Dados de Pilares:

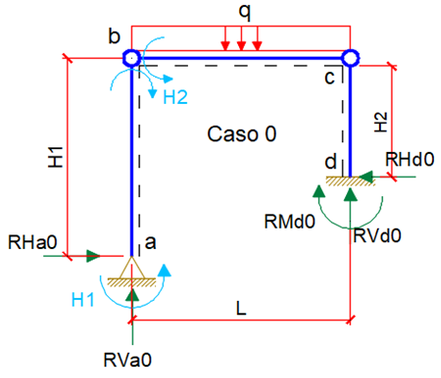
$E_pilar := 30000\ MPa$ $b_pilar := 20\ cm$ $h_pilar := 20\ cm$

- Dados da Viga:

$E_viga := 30000\ MPa$ $b_viga := 20\ cm$ $h_viga := 40\ cm$

$L := 4,0\ m$ $H1 := 5,0\ m$ $H2 := 3,0\ m$ $q := 40\ \frac{kN}{m}$

Montagem do Sistema Principal e dos Casos Auxiliares - Diagramas de Momentos Fletores dos Casos:



$RHa0 := 0$

$RHd0 := 0$

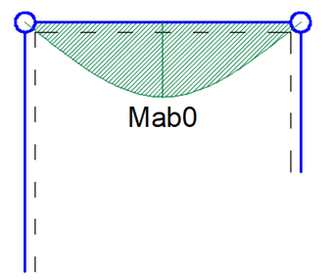
$RMd0 := 0$

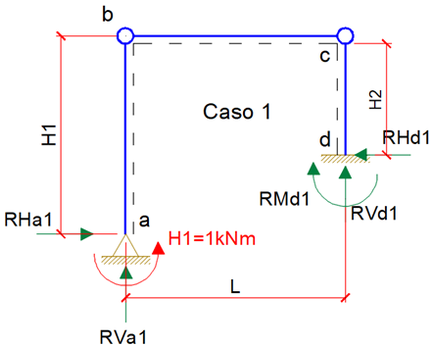
$RVa0 := q \cdot \frac{L}{2} = 80\ kN$

$RVd0 := q \cdot \frac{L}{2} = 80\ kN$

$Mab0 := \frac{q \cdot L^2}{8} = 80\ kN\ m$

M0





$RHa1 := -\frac{1\ kN\ m}{H1} = -0,2\ kN$

$RHd1 := RHa1 = -0,2\ kN$

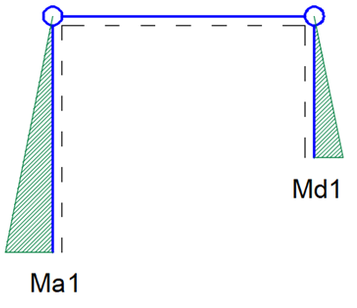
$RMd1 := -RHd1 \cdot H2 = 0,6\ kN\ m$

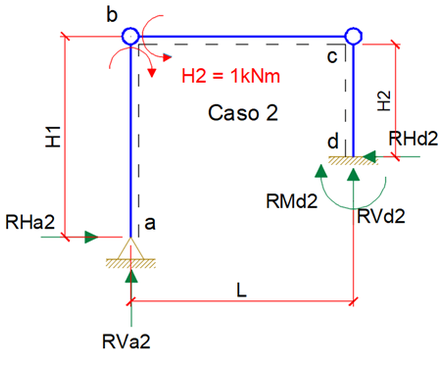
$RVa1 := 0$ $RVd1 := 0$

$Ma1 := -1\ kN\ m$

$Md1 := -RMd1 = -0,6\ kN\ m$

M1





$RHa2 := \frac{1\ kN\ m}{H1} = 0,2\ kN$

$RHd2 := RHa2 = 0,2\ kN$

$RMd2 := -RHd2 \cdot H2 = -0,6\ kN\ m$

$\Sigma Mc = 0$ (+) antihorário

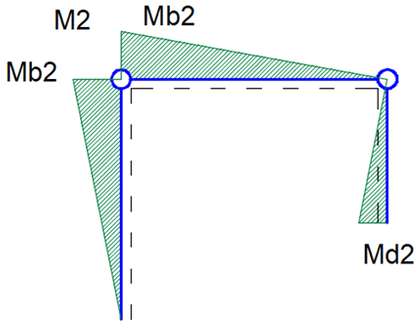
$RHa2 \cdot H1 - RVa2 \cdot L = 0$

$RVa2 := RHa2 \cdot \frac{H1}{L} = 0,25\ kN$

$RVd2 := -RVa2 = -0,25\ kN$

$Mb2 := -1\ kN\ m$ $Md2 := -RMd2 = 0,6\ kN\ m$

M2

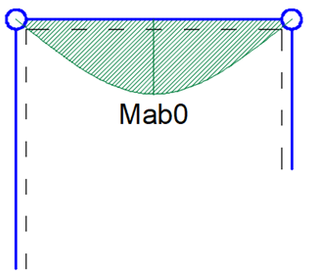


$EI_viga := \frac{E_viga \cdot b_viga \cdot h_viga^{3}}{12} = 32000\ kN\ m^{2}$

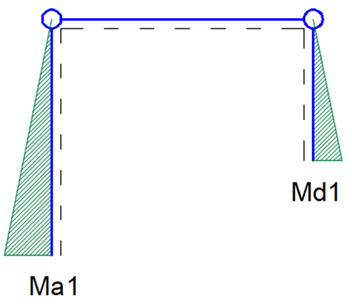
$EI_pilar := \frac{E_pilar \cdot b_pilar \cdot h_pilar^{3}}{12} = 4000\ kN\ m^{2}$ $\qquad \frac{EI_viga}{EI_pilar} = 8$

Combinando os diagramas M0 e M1:

M0



M1



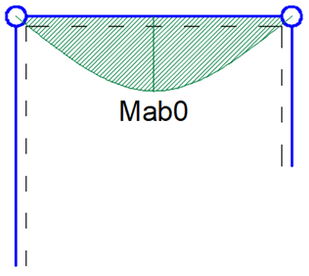
$$\varphi 10 := \frac{1}{EI} \cdot \int_{0}^{L} M0(x) \cdot M1(x)\, dx$$

Na combinação dos diagramas todas as barras acabam encontrando um resultado nulo.

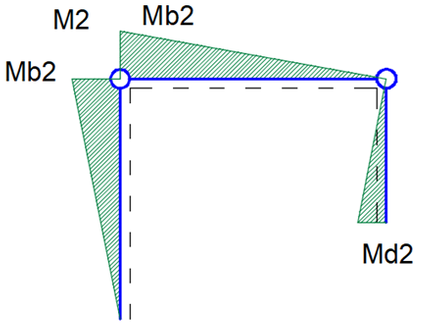
$\varphi 10 := 0$

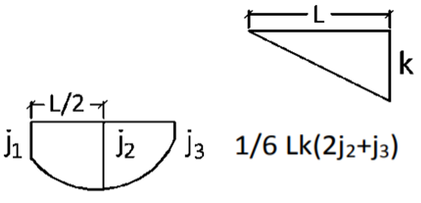
Combinando os diagramas M0 e M2:

M0



M2





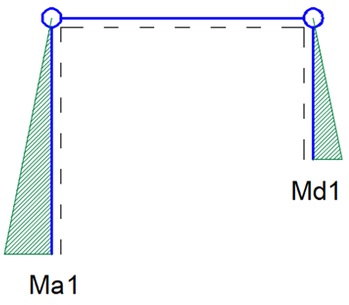
$k := Mb2 = -1\ kN\ m$ $\qquad j2 := Mab0 = 80\ kN\ m$

$j1 := 0\ kN\ m$ $\qquad j3 := 0\ kN\ m$

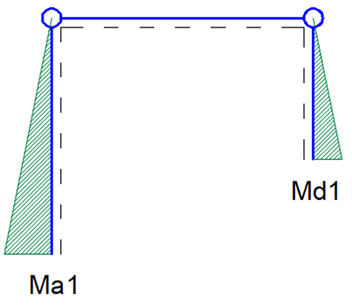
$$\varphi 20 := \frac{1}{EI_viga} \cdot \frac{1}{6} \cdot L \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{1\ kN\ m} = -0,0033333$$

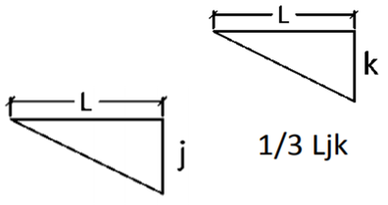
Combinando os diagramas M1 e M1:

M1



M1

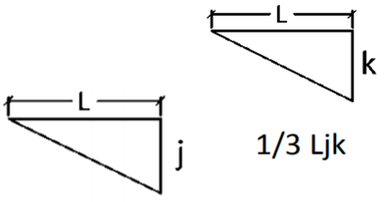




$$f11 := \frac{1}{EI} \cdot \int_{0}^{L} M1(x) \cdot M1(x)\, dx$$

$k := Ma1 = -1\ kN\ m$ $\qquad j := Ma1 = -1\ kN\ m$

$$f11ab := \frac{1}{EI_pilar} \cdot \frac{1}{3} \cdot H1 \cdot j \cdot k \cdot \frac{1}{1\ kN\ m} = 0,0004$$

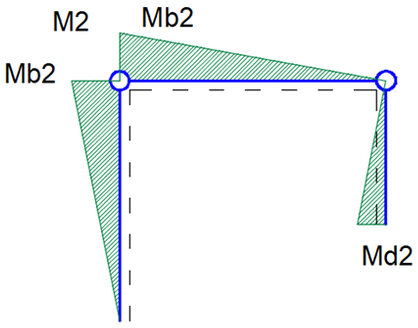


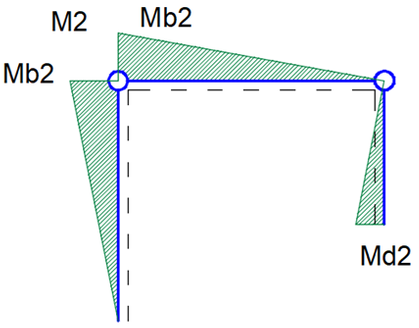
$k := Md1 = -0,6\ \text{kN m} \qquad j := Md1 = -0,6\ \text{kN m}$

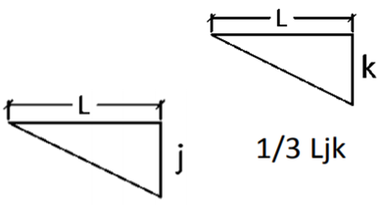
$$f11cd := \frac{1}{EI_pilar} \cdot \frac{1}{3} \cdot H2 \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = 9 \cdot 10^{-5}$$

$f11 := f11ab + f11cd = 0,00050667$

Combinando os diagramas M2 e M2:







$$f22 := \frac{1}{EI} \cdot \int_0^L M2(x) \cdot M2(x)\,dx$$

$k := Mb2 = -1\ \text{kN m} \qquad j := Mb2 = -1\ \text{kN m}$

$$f22ab := \frac{1}{EI_pilar} \cdot \frac{1}{3} \cdot H1 \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = 0,0004$$

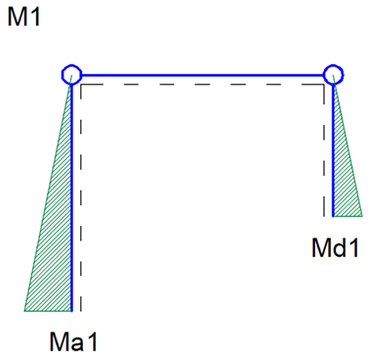
$$f22bc := \frac{1}{EI_viga} \cdot \frac{1}{3} \cdot L \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = 4,1667 \cdot 10^{-5}$$

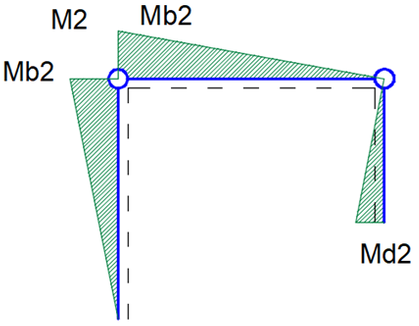
$k := Md2 = 0,6\ \text{kN m} \qquad j := Md2 = 0,6\ \text{kN m}$

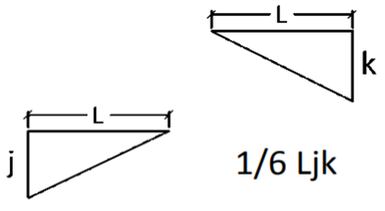
$$f22cd := \frac{1}{EI_pilar} \cdot \frac{1}{3} \cdot H2 \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = 9 \cdot 10^{-5}$$

$f22 := f22ab + f22bc + f22cd = 0,0005$

Combinando os diagramas M1 e M2:



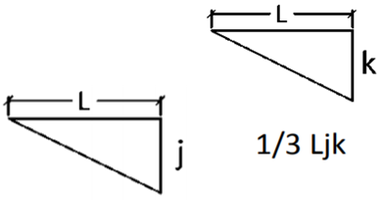




$$f12 := \frac{1}{EI} \cdot \int_0^L M1(x) \cdot M2(x)\,dx$$

$k := Mb2 = -1\ \text{kN m} \qquad j := Ma1 = -1\ \text{kN m}$

$$f12ab := \frac{1}{EI_pilar} \cdot \frac{1}{6} \cdot H1 \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = 0,0002$$



$k := Md1 = -0,6\ \text{kN m}$ $\quad j := Md2 = 0,6\ \text{kN m}$

$$f12cd := \frac{1}{EI_pilar} \cdot \frac{1}{3} \cdot H2 \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = -9 \cdot 10^{-5}$$

$f12 := f12ab + f12cd = 0,0001$

Composição do Sistema de Equações de Compatibilidade e Cálculo das Redundantes Hiperestáticas Escolhidas:

Resolvendo o sistema de equações por meio eletrônico:

$R1 \cdot f11 + R2 \cdot f12 = -\varphi 10$

$R1 \cdot f21 + R2 \cdot f22 = -\varphi 20$

$$A := \begin{bmatrix} f11 & f12 \\ f12 & f22 \end{bmatrix} \qquad b := \begin{bmatrix} -\varphi 10 \\ -\varphi 20 \end{bmatrix} \qquad R := A^{-1} \cdot b = \begin{bmatrix} -1,4951 \\ 6,4017 \end{bmatrix}$$

$Ma := R_1 = -1,4951$

$Mb := R_2 = 6,4017$

Cálculo das Demais Reações de Apoio:

Sobreposição de Efeitos:

$Fi_real = Fi0 + Fi1 \cdot R1 + Fi2 \cdot R2$ $\qquad R1 := Ma = -1,4951$ $\qquad R2 := Mb = 6,4017$

$RVa := RVa0 + RVa1 \cdot R1 + RVa2 \cdot R2 = 81,6\ \text{kN}$

$RVd := RVd0 + RVd1 \cdot R1 + RVd2 \cdot R2 = 78,3996\ \text{kN}$

$RHa := RHa0 + RHa1 \cdot R1 + RHa2 \cdot R2 = 1,5794\ \text{kN}$

$RHd := RHd0 + RHd1 \cdot R1 + RHd2 \cdot R2 = 1,5794\ \text{kN}$

$RMd := RMd0 + RMd1 \cdot R1 + RMd2 \cdot R2 = -4,7381\ \text{kN m}$
