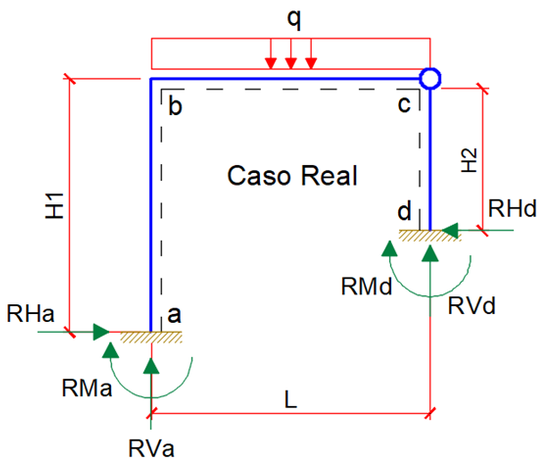
TE2_E05-02b

- Determinar as reações de apoio da estrutura hiperestática ao lado (considerando somente efeitos de flexão):



- Dados de Pilares:

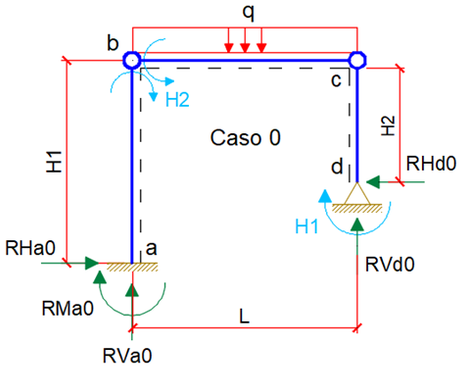
$E_pilar := 30000\ MPa$ $b_pilar := 20\ cm$ $h_pilar := 20\ cm$

- Dados da Viga:

$E_viga := 30000\ MPa$ $b_viga := 20\ cm$ $h_viga := 40\ cm$

$L := 4,0\ m$ $H1 := 5,0\ m$ $H2 := 3,0\ m$ $q := 40\ \frac{kN}{m}$

Montagem do Sistema Principal e dos Casos Auxiliares - Diagramas de Momentos Fletores dos Casos:



$RHd0 := 0\ kN$

$RHa0 := 0\ kN$

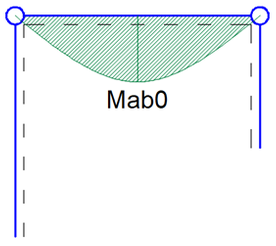
$RMa0 := 0\ kN\ m$

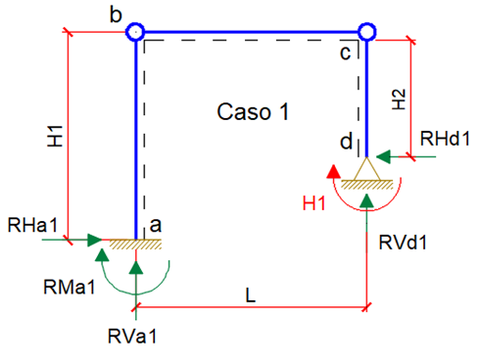
$RVa0 := q \cdot \frac{L}{2} = 80\ kN$

$RVd0 := q \cdot \frac{L}{2} = 80\ kN$

$Mab0 := \frac{q \cdot L^2}{8} = 80\ kN\ m$

M0





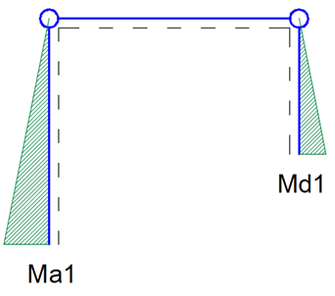
$RHd1 := -\frac{1\ kN\ m}{H2} = -0,33333\ kN$

$RHa1 := RHd1 = -0,33333\ kN$

$RMa1 := RHa1 \cdot H1 = -1,6667\ kN\ m$

$\Sigma Ma = 0$ (+) horário

M1



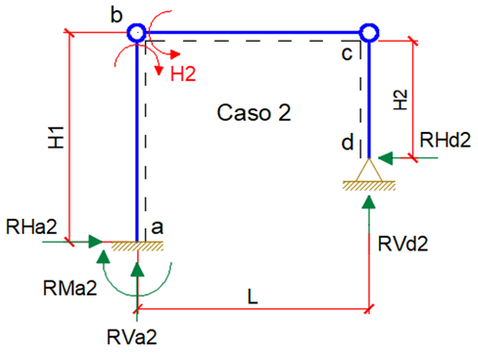
$RMa1 + 1\ kN\ m - RHd1 \cdot (H1 - H2) - RVd1 \cdot L = 0$

$RVd1 := \frac{1}{L} \cdot (RMa1 + 1\ kN\ m - RHd1 \cdot (H1 - H2)) = 0$

$RVa1 := 0$

$Ma1 := RMa1 = -1,6667\ kN\ m$

$Md1 := -1\ kN\ m$



$RHd2 := 0\ kN\ m$

$RHa2 := 0\ kN\ m$

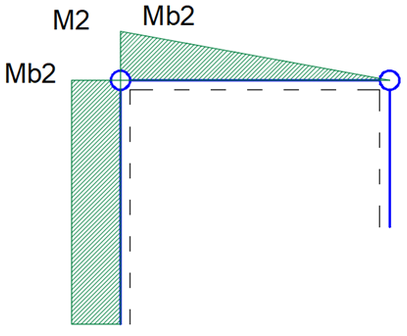
$RMa2 := -1\ kN\ m$

$RVd2 := \frac{-1\ kN\ m}{L} = -0,25\ kN$

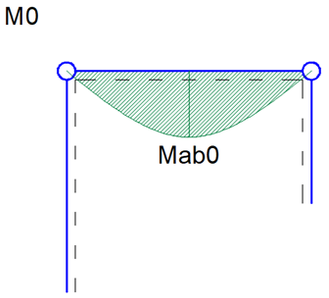
$RVa2 := -RVd2 = 0,25\ kN$

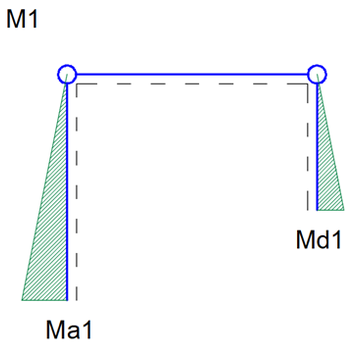
$Mb2 := -1\ kN\ m$

M2 Mb2



Combinando os Diagramas M0 e M1 para cálculo da rotação φ10:

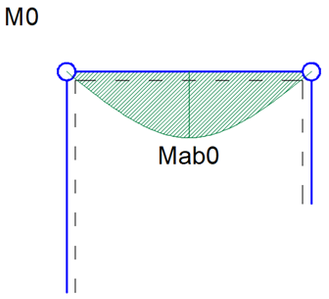


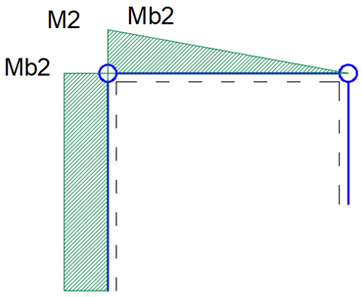


$$\varphi 10 := \frac{1}{EI} \cdot \int_0^L M1(x) \cdot M0(x)\,dx$$

$$\varphi 10 := 0$$

Combinando os Diagramas M0 e M2 para cálculo da rotação φ20:

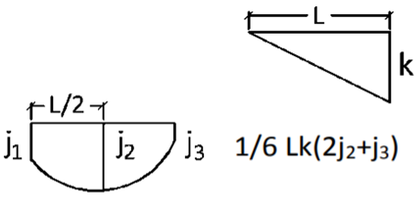




$$EI_viga := \frac{E_viga \cdot b_viga \cdot h_viga^{3}}{12}$$

$$EI_viga = 32000\ \text{kN m}^2$$

$$\varphi 20 := \frac{1}{EI} \cdot \int_0^L M2(x) \cdot M0(x)\,dx$$

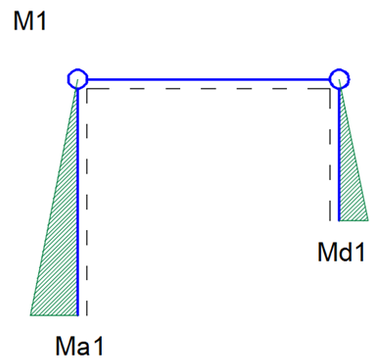


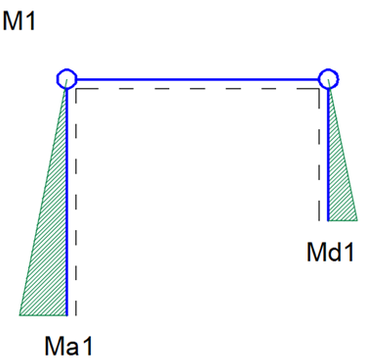
$k := Mb2 = -1\ \text{kN m}$ $\quad j2 := Mab0 = 80\ \text{kN m}$ $\quad L = 4\ \text{m}$

$j1 := 0\ \text{kN m}$ $\quad j3 := 0\ \text{kN m}$

$$\varphi 20 := \frac{1}{EI_viga} \cdot \frac{1}{6} \cdot L \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{1\ \text{kN m}} = -0,0033333$$

Combinando os diagramas M1 e M1 para cálculo de f11:

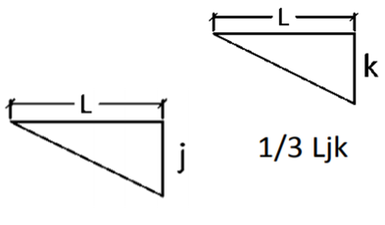




$$f11 := \frac{1}{EI} \cdot \int_0^L M1(x) \cdot M1(x)\,dx$$

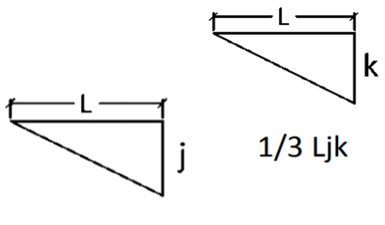
$$EI_pilar := \frac{E_pilar \cdot b_pilar \cdot h_pilar^{3}}{12}$$

$$EI_pilar = 4000\ \text{kN m}^2$$



$k := Ma1 = -1,6667\ \text{kN m}$ $\quad j := Ma1 = -1,6667\ \text{kN m}$

$$f11ab := \frac{1}{EI_pilar} \cdot \frac{1}{3} \cdot H1 \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = 0,0012$$



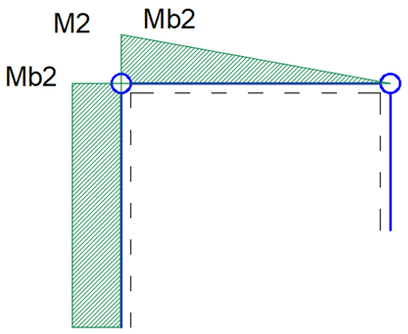
$k := Md1 = -1\ \text{kN m}$ $\quad j := Md1 = -1\ \text{kN m}$

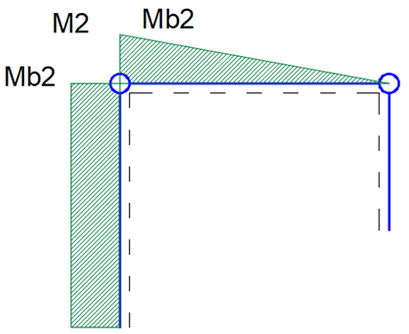
$$f11cd := \frac{1}{EI_pilar} \cdot \frac{1}{3} \cdot H2 \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = 0,0002$$

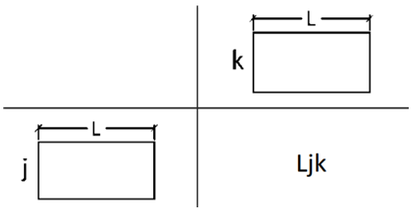
$$f11 := f11ab + f11cd = 0,0014074$$

Combinando os diagramas M2 e M2 para cálculo de f22:

$$f22 := \frac{1}{EI} \cdot \int_0^L M2(x) \cdot M2(x)\,dx$$

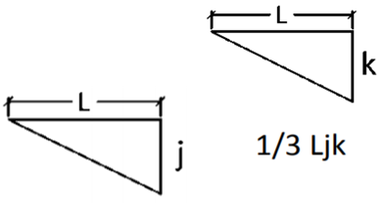






$k := Mb2 = -1\ \text{kN m}$ $\quad j := Mb2 = -1\ \text{kN m}$

$$f22ab := \frac{1}{EI_pilar} \cdot H1 \cdot j \cdot k \cdot \frac{1}{\text{kN m}} = 0,00125$$



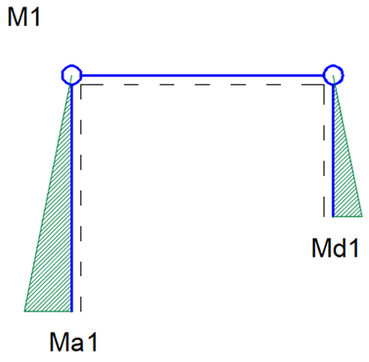
$k := Mb2 = -1\ \text{kN m}$ $\quad j := Mb2 = -1\ \text{kN m}$

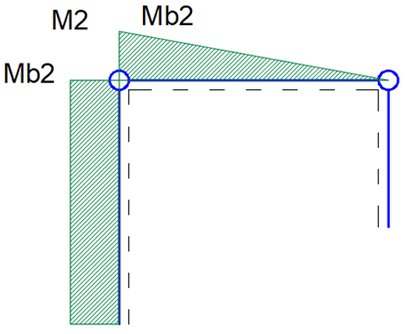
$$f22bc := \frac{1}{EI_viga} \cdot \frac{1}{3} \cdot L \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = 4,1667 \cdot 10^{-5}$$

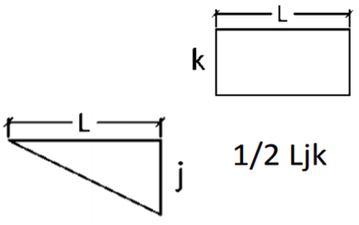
$$f22 := f22ab + f22bc = 0,0012917$$

Combinando os diagramas M1 e M2 para cálculo de f12 e f21 (iguais):

$$f12 := \frac{1}{EI} \cdot \int_0^L M1(x) \cdot M2(x)\,dx$$







$k := Mb2 = -1\ \text{kN m}$ $\quad j := Ma1 = -1,6667\ \text{kN m}$

$$f12 := \frac{1}{EI_pilar} \cdot \frac{1}{2} \cdot H1 \cdot j \cdot k \cdot \frac{1}{1\ \text{kN m}} = 0,001$$

Composição do Sistema de Equações de Compatibilidade e Cálculo das Redundantes Hiperestáticas Escolhidas:

Resolvendo o sistema de equações por meio eletrônico:

$$R1 \cdot f11 + R2 \cdot f12 = -\varphi 10$$

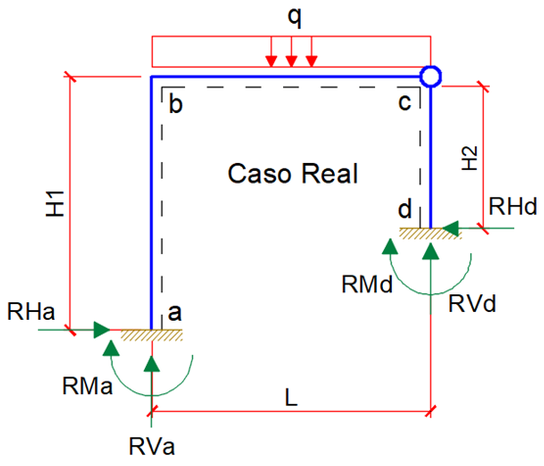
$$R1 \cdot f21 + R2 \cdot f22 = -\varphi 20$$

$$A := \begin{bmatrix} f11 & f12 \\ f12 & f22 \end{bmatrix} \qquad b := \begin{bmatrix} -\varphi 10 \\ -\varphi 20 \end{bmatrix} \qquad R := A^{-1} \cdot b = \begin{bmatrix} -4,7381 \\ 6,4017 \end{bmatrix}$$

$$RMd := R_1 = -4,7381$$

$$Mb := R_2 = 6,4017$$

Cálculo das Demais Reações de Apoio:



Sobreposição de Efeitos:

$Fi_real = Fi0 + Fi1 \cdot R1 + Fi2 \cdot R2$ $R1 := RMd = -4,7381$ $R2 := Mb = 6,4017$

$RVa := RVa0 + RVa1 \cdot R1 + RVa2 \cdot R2 = 81,6\ kN$

$RVd := RVd0 + RVd1 \cdot R1 + RVd2 \cdot R2 = 78,3996\ kN$

$RHa := RHa0 + RHa1 \cdot R1 + RHa2 \cdot R2 = 1,5794\ kN$

$RHd := RHd0 + RHd1 \cdot R1 + RHd2 \cdot R2 = 1,5794\ kN$

$RMa := RMa0 + RMa1 \cdot R1 + RMa2 \cdot R2 = 1,4951\ kN\ m$