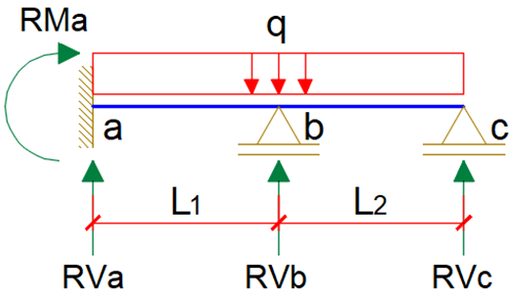
TE2_E05-05

- Determinar as reações de apoio da estrutura hiperestática ao lado (considerando somente efeitos de flexão) levando em conta que o apoio "b" sofre um deslocamento vertical prescrito (inicial) de 2 mm para baixo.



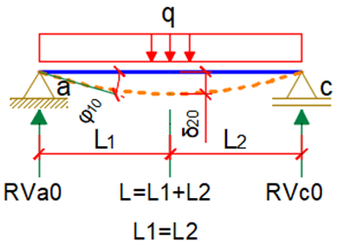
- Dados:

$E := 20000\ \text{MPa}$ $\quad L1 := 4,0\ \text{m}$ $\quad L2 := 4,0\ \text{m}$

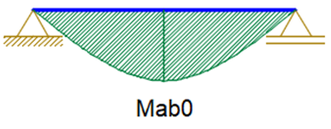
$b1 := 20\ \text{cm}$ $\quad h1 := 60\ \text{cm}$ $\quad q := 10\ \frac{\text{kN}}{\text{m}}$

Montagem do Sistema Principal e dos Casos Auxiliares - Diagramas de Momentos Fletores dos Casos:

Caso 0 (Sistema Principal)



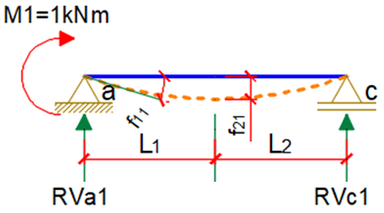
M0



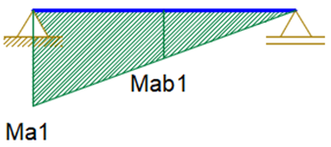
$L := L1 + L2 = 8\ \text{m}$

$Mab0 := \frac{q \cdot L^2}{8} = 80\ \text{kN m}$

Caso 1



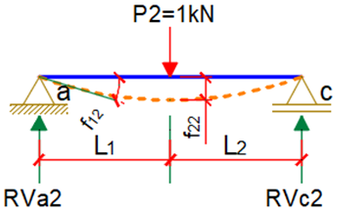
M1



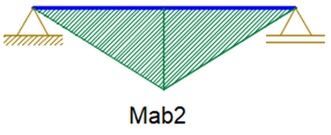
$Ma1 := 1\ \text{kN m}$

$Mab1 := \frac{Ma1}{2} = 0,5\ \text{kN m}$

Caso 2



M2



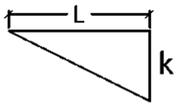
$P2 := 1\ \text{kN}$

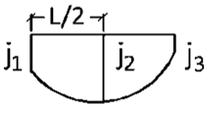
$Mab2 := \frac{P2 \cdot L1 \cdot L2}{L1 + L2} = 2\ \text{kN m}$

Combinando os diagramas M0 e M1:

$$\varphi 10 := \frac{1}{EI} \cdot \int_0^L M0(x) \cdot M1(x)\,dx$$

$EI := \frac{E \cdot b1 \cdot h1^3}{12} = 72000\ \text{kN m}^2$





1/6 Lk(2j₂+j₃)

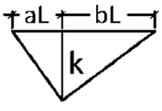
$k := Ma1 = 1\ \text{kN m}$ $\quad j2 := Mab0 = 80\ \text{kN m}$

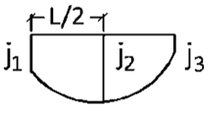
$j1 := 0\ \text{kN m}$ $\quad j3 := 0\ \text{kN m}$

$\varphi 10 := \frac{1}{EI} \cdot \frac{1}{6} \cdot L \cdot k \cdot (2 \cdot j2 + j3) \cdot \frac{1}{1\ \text{kN}} = 0,0029630\ \text{m}$

Combinando os diagramas M0 e M2:

$$\delta 20 := \frac{1}{EI} \cdot \int_0^L M0(x) \cdot M2(x)\,dx$$





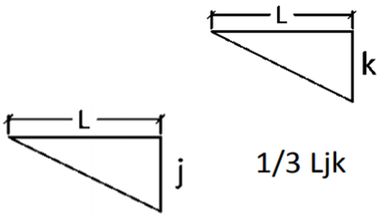
1/6 Lk[j₁b+2j₂+j₃a -ab(j₁-2j₂+j₃)]

$a := 0,5$ $\quad b := 0,5$ $\quad L = 8\ \text{m}$ $\quad k := Mab2 = 2\ \text{kN m}$

$\delta 20 := \frac{1}{EI} \cdot \frac{1}{6} \cdot L \cdot k \cdot (j1 \cdot b + 2 \cdot j2 + j3 \cdot a - a \cdot b \cdot (j1 - 2 \cdot j2 + j3)) \cdot \frac{1}{\text{kN}}$

$\delta 20 = 0,0074074\ \text{m}$

Combinando os diagramas M1 e M1:



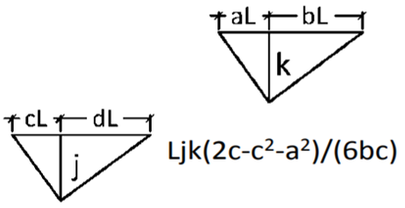
1/3 Ljk

$$f11 := \frac{1}{EI} \cdot \int_0^L M1(x) \cdot M1(x)\,dx$$

$k := Ma1 = 1\ kN\ m \qquad j := Ma1 = 1\ kN\ m$

$$f11 := \frac{1}{EI} \cdot \frac{1}{3} \cdot L \cdot j \cdot k \cdot \frac{1}{1\ kN} = 3{,}7037 \cdot 10^{-5}\ m$$

Combinando os diagramas M2 e M2:



Ljk(2c-c²-a²)/(6bc)

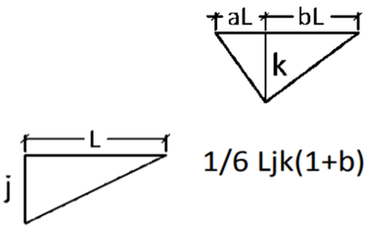
$$f22 := \frac{1}{EI} \cdot \int_0^L M2(x) \cdot M2(x)\,dx$$

$c := a = 0{,}5 \qquad d := b = 0{,}5$

$k := Mab2 = 2\ kN\ m \qquad j := Mab2 = 2\ kN\ m$

$$f22 := \frac{1}{EI} \cdot L \cdot j \cdot k \cdot \frac{\left(2 \cdot c - c^2 - a^2\right)}{6 \cdot b \cdot c} \cdot \frac{1}{1\ kN} = 0{,}0001\ m$$

Combinando os diagramas M1 e M2:



1/6 Ljk(1+b)

$$f12 := \int_0^L M1(x) \cdot M2(x)\,dx$$

$a = 0{,}5 \qquad b = 0{,}5 \qquad k := Mab2 = 2\ kN\ m \qquad j := Ma1 = 1\ kN\ m$

$$f12 := \frac{1}{EI} \cdot \frac{1}{6} \cdot L \cdot j \cdot k \cdot (1 + b) \cdot \frac{1}{1\ kN} = 5{,}5556 \cdot 10^{-5}\ m$$

Composição do Sistema de Equações de Compatibilidade e Cálculo das Redundantes Hiperestáticas Escolhidas:

Resolvendo o sistema de equações por meio eletrônico:

$$Solu_Sem_Desloc_Presc := \begin{bmatrix} -11{,}4286 \\ -45{,}7143 \end{bmatrix}$$

- φ1_real e δ2_real são os deslocamentos prescritos na Estrutura Real

$\varphi1_real := 0 \qquad \delta2_real := 0{,}002\ m$

- Como definir o sinal de φ1_real e δ2_real ?

$R1 \cdot f11 + R2 \cdot f12 + \varphi10 = \varphi1_real$

$R1 \cdot f21 + R2 \cdot f22 + \delta20 = \delta2_real$

---> [A] * {R} + {δ0} = {δR}

[A] : matriz dos coeficientes de flexibilidade;
{R} : vetor das redudantes hiperestáticas;
{δ0} : deslocamentos no caso 0 nos pontos das redundantes hiperestáticas;
{δR} : deslocamentos prescritos na estrutura real, nos vínculos escolhidos como redundantes hiperestáticas;

*** Dica: caso existam deslocamentos prescritos em vínculos, escolher estes vínculos como redundantes hiperestáticas;

---> [A] * {R} + {δ0} = {δR} ---> [A] * {R} = {δR} - {δ0} ---> {R} = [A]^(-1) * ({δR} - {δ0})

$$A := \begin{bmatrix} f11 & f12 \\ f12 & f22 \end{bmatrix} \qquad \delta0 := \begin{bmatrix} \varphi10 \\ \delta20 \end{bmatrix} \qquad R_sem_Desloc := A^{-1} \cdot (-\delta0) = \begin{bmatrix} -11{,}4286 \\ -45{,}7143 \end{bmatrix}$$

$$\delta R := \begin{bmatrix} \varphi1_real \\ \delta2_real \end{bmatrix} \qquad R_com_Desloc := A^{-1} \cdot (\delta R - \delta0) = \begin{bmatrix} -57{,}7143 \\ -14{,}8571 \end{bmatrix}$$

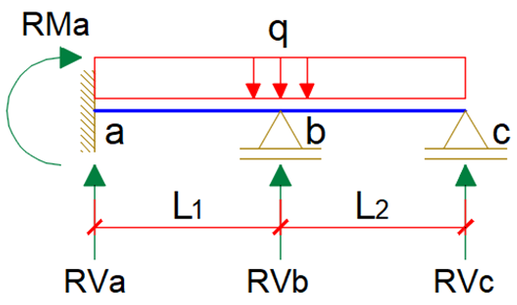
$Ma_sem_Desloc := R_sem_Desloc_{1\,1}\ kN\ m = -11{,}429\ kN\ m$ (gira anti-horário)

$RVb_sem_Desloc := R_sem_Desloc_{2\,1}\ kN = -45{,}7143\ kN$ (reação vertical subindo)

$Ma_com_Desloc := R_com_Desloc_{1\,1}$ kN m $= -57,7143$ kN m (gira anti-horário)

$RVb_com_Desloc := R_com_Desloc_{2\,1}$ kN $= -14,8571$ kN (reação vertical subindo)

Cálculo das Demais Reações de Apoio:



$RVb := -RVb_com_Desloc = 14,8571$ kN

$RMa := Ma_com_Desloc = -57,7143$ kN m

Cálculo de RVa: $\Sigma Mc = 0$ (+) horário

$$RVa \cdot L + RMa + RVb \cdot L2 - \frac{q \cdot L^2}{2} = 0$$

$$RVa := \frac{1}{L} \cdot \left(-RMa - RVb \cdot L2 + \frac{q \cdot L^2}{2} \right) = 39,7857 \text{ kN}$$

Cálculo de RVc: $\Sigma Fv = 0$ (+) para cima

$$RVa + RVb + RVc - q \cdot L = 0$$

$RVc := -RVa - RVb + q \cdot L = 25,3571$ kN
