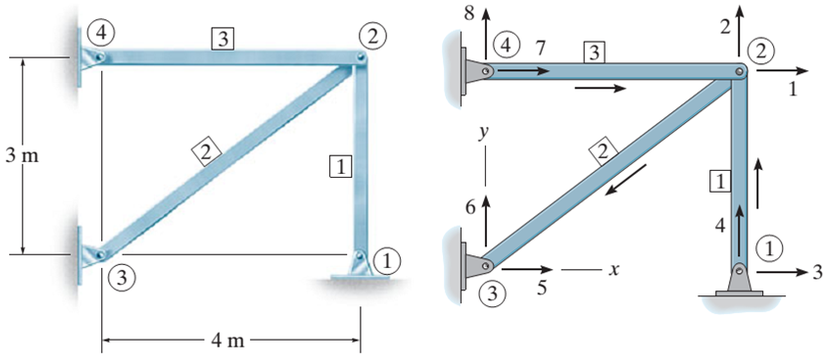
TE2_E08-02

Determinar o esforço axial na barra [2] da estrutura ao lado quando o nó (1) sofre um deslocamento vertical descendo 25 mm.



Todas as barras possuem o mesmo EA :

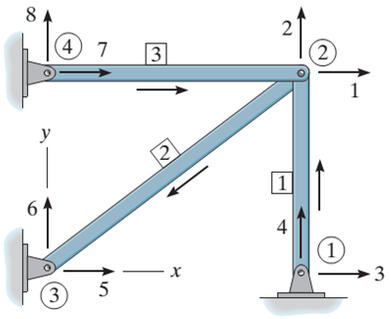
$EA := 8000\ \text{kN}$

- Dados:

$L := 4,0\ \text{m}$ $H := 3,0\ \text{m}$

Matrizes de Rigidez Elementares em Coordenadas Globais de Cada Barra:

Coordenadas (origem do sistema de eixos global no ponto "3"):



$x3 := 0$ $y3 := 0$

$x1 := L = 4$ $y1 := 0$

$x2 := L = 4$ $y2 := H = 3$

$x4 := 0$ $y4 := H = 3$

$$[k] = \frac{AE}{L}\begin{bmatrix} \lambda_x^2 & \lambda_x\lambda_y & -\lambda_x^2 & -\lambda_x\lambda_y \\ \lambda_x\lambda_y & \lambda_y^2 & -\lambda_x\lambda_y & -\lambda_y^2 \\ -\lambda_x^2 & -\lambda_x\lambda_y & \lambda_x^2 & \lambda_x\lambda_y \\ -\lambda_x\lambda_y & -\lambda_y^2 & \lambda_x\lambda_y & \lambda_y^2 \end{bmatrix} \begin{matrix} N_x \\ N_y \\ F_x \\ F_y \end{matrix}$$

Matriz de Rigidez Elementar em Coord. Globais da Barra 1:

$L1 := \sqrt{(x2 - x1)^2 + (y2 - y1)^2} = 3$ $\lambda x1 := \frac{x2 - x1}{L1} = 0$ $\lambda y1 := \frac{y2 - y1}{L1} = 1$

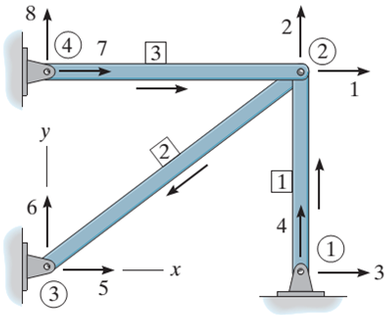
$$k1 := \frac{EA}{L1} \cdot \begin{bmatrix} \lambda x1^2 & \lambda x1 \cdot \lambda y1 & -\lambda x1^2 & -\lambda x1 \cdot \lambda y1 \\ \lambda x1 \cdot \lambda y1 & \lambda y1^2 & -\lambda x1 \cdot \lambda y1 & -\lambda y1^2 \\ -\lambda x1^2 & -\lambda x1 \cdot \lambda y1 & \lambda x1^2 & \lambda x1 \cdot \lambda y1 \\ -\lambda x1 \cdot \lambda y1 & -\lambda y1^2 & \lambda x1 \cdot \lambda y1 & \lambda y1^2 \end{bmatrix} = \begin{matrix} [3\ 4\ 1\ 2] \\ \begin{bmatrix} 0,000 & 0,000 & 0,000 & 0,000 \\ 0,000 & 2667 & 0,000 & -2667 \\ 0,000 & 0,000 & 0,000 & 0,000 \\ 0,000 & -2667 & 0,000 & 2667 \end{bmatrix} \end{matrix} \begin{bmatrix} 3 \\ 4 \\ 1 \\ 2 \end{bmatrix}$$

Matriz de Rigidez Elementar em Coord. Globais da Barra 2:

$L2 := \sqrt{(x3 - x2)^2 + (y3 - y2)^2} = 5$ $\lambda x2 := \frac{x3 - x2}{L2} = -\frac{4}{5}$ $\lambda y2 := \frac{y3 - y2}{L2} = -\frac{3}{5}$

$$k2 := \frac{EA}{L2} \cdot \begin{bmatrix} \lambda x2^2 & \lambda x2 \cdot \lambda y2 & -\lambda x2^2 & -\lambda x2 \cdot \lambda y2 \\ \lambda x2 \cdot \lambda y2 & \lambda y2^2 & -\lambda x2 \cdot \lambda y2 & -\lambda y2^2 \\ -\lambda x2^2 & -\lambda x2 \cdot \lambda y2 & \lambda x2^2 & \lambda x2 \cdot \lambda y2 \\ -\lambda x2 \cdot \lambda y2 & -\lambda y2^2 & \lambda x2 \cdot \lambda y2 & \lambda y2^2 \end{bmatrix} = \begin{matrix} [1\ 2\ 5\ 6] \\ \begin{bmatrix} 1024 & 768 & -1024 & -768 \\ 768 & 576 & -768 & -576 \\ -1024 & -768 & 1024 & 768 \\ -768 & -576 & 768 & 576 \end{bmatrix} \end{matrix} \begin{bmatrix} 1 \\ 2 \\ 5 \\ 6 \end{bmatrix}$$

Matriz de Rigidez Elementar em Coord. Globais da Barra 3:



$$L3 := \sqrt{(x2 - x4)^2 + (y2 - y4)^2} = 4$$

$$\lambda x3 := \frac{x2 - x4}{L3} = 1 \qquad \lambda y3 := \frac{y2 - y4}{L3} = 0$$

$$k3 := \frac{EA}{L3} \cdot \begin{bmatrix} \lambda x3^2 & \lambda x3 \cdot \lambda y3 & -\lambda x3^2 & -\lambda x3 \cdot \lambda y3 \\ \lambda x3 \cdot \lambda y3 & \lambda y3^2 & -\lambda x3 \cdot \lambda y3 & -\lambda y3^2 \\ -\lambda x3^2 & -\lambda x3 \cdot \lambda y3 & \lambda x3^2 & \lambda x3 \cdot \lambda y3 \\ -\lambda x3 \cdot \lambda y3 & -\lambda y3^2 & \lambda x3 \cdot \lambda y3 & \lambda y3^2 \end{bmatrix} = \begin{bmatrix} 2000 & 0 & -2000 & 0 \\ 0 & 0 & 0 & 0 \\ -2000 & 0 & 2000 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} [7\ 8\ 1\ 2] \\ \begin{bmatrix} 7 \\ 8 \\ 1 \\ 2 \end{bmatrix} \end{matrix}$$

Montagem da Matriz de Rigidez da Estrutura como um Todo:

$$KG := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Espalhamento de "k1": $[3\ 4\ 1\ 2]$

$$k1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2666,7 & 0 & -2666,7 \\ 0 & 0 & 0 & 0 \\ 0 & -2666,7 & 0 & 2666,7 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \\ 2 \end{bmatrix}$$

$$KG := \begin{bmatrix} k1_{3\,3} & k1_{3\,4} & k1_{3\,1} & k1_{3\,2} & 0 & 0 & 0 & 0 \\ k1_{4\,3} & k1_{4\,4} & k1_{4\,1} & k1_{4\,2} & 0 & 0 & 0 & 0 \\ k1_{1\,3} & k1_{1\,4} & k1_{1\,1} & k1_{1\,2} & 0 & 0 & 0 & 0 \\ k1_{2\,3} & k1_{2\,4} & k1_{2\,1} & k1_{2\,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2666,7 & 0 & -2666,7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2666,7 & 0 & 2666,7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Espalhamento de "k2": $[1\ 2\ 5\ 6]$

$$k2 = \begin{bmatrix} 1024 & 768 & -1024 & -768 \\ 768 & 576 & -768 & -576 \\ -1024 & -768 & 1024 & 768 \\ -768 & -576 & 768 & 576 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 5 \\ 6 \end{bmatrix}$$

$$KG := \begin{bmatrix} k1_{3\,3} + k2_{1\,1} & k1_{3\,4} + k2_{1\,2} & k1_{3\,1} & k1_{3\,2} & k2_{1\,3} & k2_{1\,4} & 0 & 0 \\ k1_{4\,3} + k2_{2\,1} & k1_{4\,4} + k2_{2\,2} & k1_{4\,1} & k1_{4\,2} & k2_{2\,3} & k2_{2\,4} & 0 & 0 \\ k1_{1\,3} & k1_{1\,4} & k1_{1\,1} & k1_{1\,2} & 0 & 0 & 0 & 0 \\ k1_{2\,3} & k1_{2\,4} & k1_{2\,1} & k1_{2\,2} & 0 & 0 & 0 & 0 \\ k2_{3\,1} & k2_{3\,2} & 0 & 0 & k2_{3\,3} & k2_{3\,4} & 0 & 0 \\ k2_{4\,1} & k2_{4\,2} & 0 & 0 & k2_{4\,3} & k2_{4\,4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1024 & 768 & 0 & 0 & -1024 & -768 & 0 & 0 \\ 768 & 3242,7 & 0 & -2666,7 & -768 & -576 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2666,7 & 0 & 2666,7 & 0 & 0 & 0 & 0 \\ -1024 & -768 & 0 & 0 & 1024 & 768 & 0 & 0 \\ -768 & -576 & 0 & 0 & 768 & 576 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Espalhamento de "k3": [7 8 1 2]

$$k3 = \begin{bmatrix} 2000 & 0 & -2000 & 0 \\ 0 & 0 & 0 & 0 \\ -2000 & 0 & 2000 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 7 \\ 8 \\ 1 \\ 2 \end{bmatrix}$$

$$KG := \begin{bmatrix} k1_{33}+k2_{11}+k3_{33} & k1_{34}+k2_{12}+k3_{34} & k1_{31} & k1_{32} & k2_{13} & k2_{14} & k3_{31} & k3_{32} \\ k1_{43}+k2_{21}+k3_{43} & k1_{44}+k2_{22}+k3_{44} & k1_{41} & k1_{42} & k2_{23} & k2_{24} & k3_{41} & k3_{42} \\ k1_{13} & k1_{14} & k1_{11} & k1_{12} & 0 & 0 & 0 & 0 \\ k1_{23} & k1_{24} & k1_{21} & k1_{22} & 0 & 0 & 0 & 0 \\ k2_{31} & k2_{32} & 0 & 0 & k2_{33} & k2_{34} & 0 & 0 \\ k2_{41} & k2_{42} & 0 & 0 & k2_{43} & k2_{44} & 0 & 0 \\ k3_{13} & k3_{14} & 0 & 0 & 0 & 0 & k3_{11} & k3_{12} \\ k3_{23} & k3_{24} & 0 & 0 & 0 & 0 & k3_{21} & k3_{22} \end{bmatrix}$$

$$KG = \begin{bmatrix} 3024 & 768 & 0 & 0 & -1024 & -768 & -2000 & 0 \\ 768 & 3242,66667 & 0 & -2666,66667 & -768 & -576 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2666,66667 & 0 & 2666,66667 & 0 & 0 & 0 & 0 \\ -1024 & -768 & 0 & 0 & 1024 & 768 & 0 & 0 \\ -768 & -576 & 0 & 0 & 768 & 576 & 0 & 0 \\ -2000 & 0 & 0 & 0 & 0 & 0 & 2000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K11 := \begin{bmatrix} KG_{11} & KG_{12} \\ KG_{21} & KG_{22} \end{bmatrix} = \begin{bmatrix} 3024 & 768 \\ 768 & 3242,7 \end{bmatrix} \qquad K12 := \begin{bmatrix} KG_{13} & KG_{14} & KG_{15} & KG_{16} & KG_{17} & KG_{18} \\ KG_{23} & KG_{24} & KG_{25} & KG_{26} & KG_{27} & KG_{28} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1024 & -768 & -2000 & 0 \\ 0 & -2666,7 & -768 & -576 & 0 & 0 \end{bmatrix}$$

$$K21 := \begin{bmatrix} KG_{31} & KG_{32} \\ KG_{41} & KG_{42} \\ KG_{51} & KG_{52} \\ KG_{61} & KG_{62} \\ KG_{71} & KG_{72} \\ KG_{81} & KG_{82} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -2666,7 \\ -1024 & -768 \\ -768 & -576 \\ -2000 & 0 \\ 0 & 0 \end{bmatrix} \qquad K22 := \begin{bmatrix} KG_{33} & KG_{34} & KG_{35} & KG_{36} & KG_{37} & KG_{38} \\ KG_{43} & KG_{44} & KG_{45} & KG_{46} & KG_{47} & KG_{48} \\ KG_{53} & KG_{54} & KG_{55} & KG_{56} & KG_{57} & KG_{58} \\ KG_{63} & KG_{64} & KG_{65} & KG_{66} & KG_{67} & KG_{68} \\ KG_{73} & KG_{74} & KG_{75} & KG_{76} & KG_{77} & KG_{78} \\ KG_{83} & KG_{84} & KG_{85} & KG_{86} & KG_{87} & KG_{88} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2666,7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1024 & 768 & 0 & 0 \\ 0 & 0 & 768 & 576 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Qk := \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

* Deslocamento prescrito de 25 mm descendo no grau de liberdade 4:

$$Qu := \begin{bmatrix} Q3 \\ Q4 \\ Q5 \\ Q6 \\ Q7 \\ Q8 \end{bmatrix} \qquad Dk := \begin{bmatrix} 0 \\ -0,025 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solução do Sistema de Equações:

Solução do Sistema de Equações:

Qk = K11*Du + K12*Dk (1ª Linha)
Qu = K21*Du + K22*Dk (2ª Linha)

Resolvendo a 1ª Linha: Du = K11^-1 * (Qk - K12*Dk)

$$Du := K11^{-1} \cdot (Qk - K12 \cdot Dk) = \begin{bmatrix} 0,0055556 \\ -0,021875 \end{bmatrix}$$

$D1 := Du_1 = 0,0056$ $D2 := Du_2 = -0,0219$ $D3 := 0$ $D4 := -0,025$ $D5 := 0$ $D6 := 0$ $D7 := 0$ $D8 := 0$

Resolvendo a 2ª Linha: Qu = K21*Du + K22*Dk

$$Qu := K21 \cdot Du + K22 \cdot Dk = \begin{bmatrix} 0 \\ -8,3333 \\ 11,1111 \\ 8,3333 \\ -11,1111 \\ 0 \end{bmatrix}$$

Reações de apoio nos graus de liberdade 3, 4, 5, 6, 7 e 8 respectivamente

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Esforços Axiais nas Barras:

$$q_F = \frac{AE}{L}[-\lambda_x \quad -\lambda_y \quad \lambda_x \quad \lambda_y] \begin{Bmatrix} D_{Nx} \\ D_{Ny} \\ D_{Fx} \\ D_{Fy} \end{Bmatrix}$$

Na Barra 2:

$$N2 := \frac{EA}{L2} \cdot \left(\left([\,-\lambda x2 \;\; -\lambda y2 \;\; \lambda x2 \;\; \lambda y2\,] \cdot \begin{bmatrix} D1 \\ D2 \\ D5 \\ D6 \end{bmatrix}^{[1\ 2\ 5\ 6]} \right)_1 \right) = -13,8889$$
