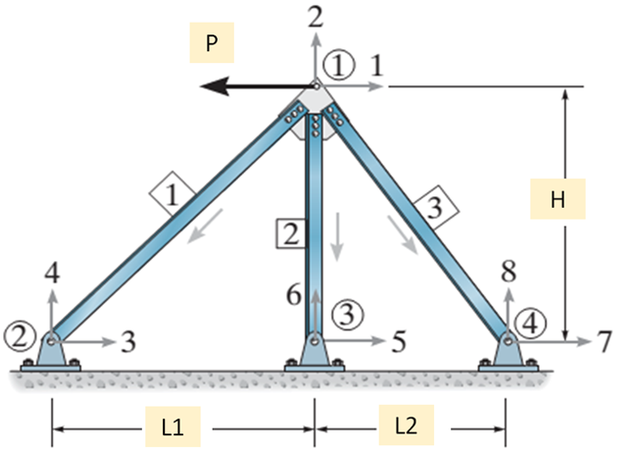
TE2_E08-03

Determinar o esforço axial na barra [2] se a sua temperatura for aumentada em 55°C.



Para todas as barras:

$E := 200$ GPa $\quad A := 1000$ mm²

$\alpha := 11,7 \cdot 10^{-6}$ /°C

- Dados:

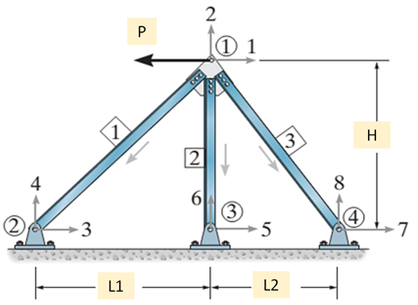
$P := 2$ kN $\quad H := 4,0$ m $\quad \Delta T := +55$ °C

$L1 := 4,0$ m $\quad L2 := 3,0$ m

Matrizes de Rigidez Elementares em Coordenadas Globais de Cada Barra:

$E := 200 \cdot 10^{6}$ kN/m² $\quad A := 1000\ \text{mm}^2 = 0,001\ \text{m}^2 \quad A := 0,001\ m^2$

Coordenadas (origem do sistema de eixos global no ponto "2"):



$x1 := L1 = 4 \quad y1 := H = 4$

$x2 := 0 = 0 \quad y2 := 0$

$x3 := L1 = 4 \quad y3 := 0 = 0$

$x4 := L1 + L2 = 7 \quad y4 := 0 = 0$

$$[k] = \frac{AE}{L}\begin{bmatrix} \lambda_x^2 & \lambda_x\lambda_y & -\lambda_x^2 & -\lambda_x\lambda_y \\ \lambda_x\lambda_y & \lambda_y^2 & -\lambda_x\lambda_y & -\lambda_y^2 \\ -\lambda_x^2 & -\lambda_x\lambda_y & \lambda_x^2 & \lambda_x\lambda_y \\ -\lambda_x\lambda_y & -\lambda_y^2 & \lambda_x\lambda_y & \lambda_y^2 \end{bmatrix} \begin{matrix} N_x \\ N_y \\ F_x \\ F_y \end{matrix}$$

Matriz de Rigidez Elementar em Coord. Globais da Barra 1:

$L1 := \sqrt{(x2 - x1)^2 + (y2 - y1)^2} = 5,6569 \quad \lambda x1 := \frac{x2 - x1}{L1} = -0,7071 \quad \lambda y1 := \frac{y2 - y1}{L1} = -0,7071$

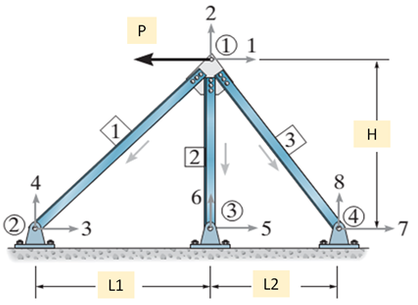
$$k1 := \frac{E \cdot A}{L1} \cdot \begin{bmatrix} \lambda x1^2 & \lambda x1 \cdot \lambda y1 & -\lambda x1^2 & -\lambda x1 \cdot \lambda y1 \\ \lambda x1 \cdot \lambda y1 & \lambda y1^2 & -\lambda x1 \cdot \lambda y1 & -\lambda y1^2 \\ -\lambda x1^2 & -\lambda x1 \cdot \lambda y1 & \lambda x1^2 & \lambda x1 \cdot \lambda y1 \\ -\lambda x1 \cdot \lambda y1 & -\lambda y1^2 & \lambda x1 \cdot \lambda y1 & \lambda y1^2 \end{bmatrix} = \begin{matrix} [1\ 2\ 3\ 4] \\ \begin{bmatrix} 17680 & 17680 & -17680 & -17680 \\ 17680 & 17680 & -17680 & -17680 \\ -17680 & -17680 & 17680 & 17680 \\ -17680 & -17680 & 17680 & 17680 \end{bmatrix} \end{matrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Matriz de Rigidez Elementar em Coord. Globais da Barra 2:

$L2 := \sqrt{(x3 - x1)^2 + (y3 - y1)^2} = 4 \quad \lambda x2 := \frac{x3 - x1}{L2} = 0 \quad \lambda y2 := \frac{y3 - y1}{L2} = -1$

$$k2 := \frac{E \cdot A}{L2} \cdot \begin{bmatrix} \lambda x2^2 & \lambda x2 \cdot \lambda y2 & -\lambda x2^2 & -\lambda x2 \cdot \lambda y2 \\ \lambda x2 \cdot \lambda y2 & \lambda y2^2 & -\lambda x2 \cdot \lambda y2 & -\lambda y2^2 \\ -\lambda x2^2 & -\lambda x2 \cdot \lambda y2 & \lambda x2^2 & \lambda x2 \cdot \lambda y2 \\ -\lambda x2 \cdot \lambda y2 & -\lambda y2^2 & \lambda x2 \cdot \lambda y2 & \lambda y2^2 \end{bmatrix} = \begin{matrix} [1\ 2\ 5\ 6] \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 50000 & 0 & -50000 \\ 0 & 0 & 0 & 0 \\ 0 & -50000 & 0 & 50000 \end{bmatrix} \end{matrix} \begin{bmatrix} 1 \\ 2 \\ 5 \\ 6 \end{bmatrix}$$

Matriz de Rigidez Elementar em Coord. Globais da Barra 3:



$$L3 := \sqrt{(x4 - x1)^2 + (y4 - y1)^2} = 5$$

$$\lambda x3 := \frac{x4 - x1}{L3} = 0,6 \qquad \lambda y3 := \frac{y4 - y1}{L3} = -0,8$$

$$k3 := \frac{E \cdot A}{L3} \cdot \begin{bmatrix} \lambda x3^2 & \lambda x3 \cdot \lambda y3 & -\lambda x3^2 & -\lambda x3 \cdot \lambda y3 \\ \lambda x3 \cdot \lambda y3 & \lambda y3^2 & -\lambda x3 \cdot \lambda y3 & -\lambda y3^2 \\ -\lambda x3^2 & -\lambda x3 \cdot \lambda y3 & \lambda x3^2 & \lambda x3 \cdot \lambda y3 \\ -\lambda x3 \cdot \lambda y3 & -\lambda y3^2 & \lambda x3 \cdot \lambda y3 & \lambda y3^2 \end{bmatrix} = \begin{matrix} [1\ 2\ 7\ 8] \\ \begin{bmatrix} 14400 & -19200 & -14400 & 19200 \\ -19200 & 25600 & 19200 & -25600 \\ -14400 & 19200 & 14400 & -19200 \\ 19200 & -25600 & -19200 & 25600 \end{bmatrix} \end{matrix} \begin{bmatrix} 1 \\ 2 \\ 7 \\ 8 \end{bmatrix}$$

Montagem da Matriz de Rigidez da Estrutura como um Todo:

$$KG := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Espalhamento de "k1":

$$k1 = \begin{matrix} [1\ 2\ 3\ 4] \\ \begin{bmatrix} 17678 & 17678 & -17678 & -17678 \\ 17678 & 17678 & -17678 & -17678 \\ -17678 & -17678 & 17678 & 17678 \\ -17678 & -17678 & 17678 & 17678 \end{bmatrix} \end{matrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$KG := \begin{bmatrix} k1_{1,1} & k1_{1,2} & k1_{1,3} & k1_{1,4} & 0 & 0 & 0 & 0 \\ k1_{2,1} & k1_{2,2} & k1_{2,3} & k1_{2,4} & 0 & 0 & 0 & 0 \\ k1_{3,1} & k1_{3,2} & k1_{3,3} & k1_{3,4} & 0 & 0 & 0 & 0 \\ k1_{4,1} & k1_{4,2} & k1_{4,3} & k1_{4,4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 17678 & 17678 & -17678 & -17678 & 0 & 0 & 0 & 0 \\ 17678 & 17678 & -17678 & -17678 & 0 & 0 & 0 & 0 \\ -17678 & -17678 & 17678 & 17678 & 0 & 0 & 0 & 0 \\ -17678 & -17678 & 17678 & 17678 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Espalhamento de "k2":

$$k2 = \begin{matrix} [1\ 2\ 5\ 6] \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 50000 & 0 & -50000 \\ 0 & 0 & 0 & 0 \\ 0 & -50000 & 0 & 50000 \end{bmatrix} \end{matrix} \begin{bmatrix} 1 \\ 2 \\ 5 \\ 6 \end{bmatrix}$$

$$KG := \begin{bmatrix} k1_{1,1}+k2_{1,1} & k1_{1,2}+k2_{1,2} & k1_{1,3} & k1_{1,4} & k2_{1,3} & k2_{1,4} & 0 & 0 \\ k1_{2,1}+k2_{2,1} & k1_{2,2}+k2_{2,2} & k1_{2,3} & k1_{2,4} & k2_{2,3} & k2_{2,4} & 0 & 0 \\ k1_{3,1} & k1_{3,2} & k1_{3,3} & k1_{3,4} & 0 & 0 & 0 & 0 \\ k1_{4,1} & k1_{4,2} & k1_{4,3} & k1_{4,4} & 0 & 0 & 0 & 0 \\ k2_{3,1} & k2_{3,2} & 0 & 0 & k2_{3,3} & k2_{3,4} & 0 & 0 \\ k2_{4,1} & k2_{4,2} & 0 & 0 & k2_{4,3} & k2_{4,4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 17678 & 17678 & -17678 & -17678 & 0 & 0 & 0 & 0 \\ 17678 & 67678 & -17678 & -17678 & 0 & -50000 & 0 & 0 \\ -17678 & -17678 & 17678 & 17678 & 0 & 0 & 0 & 0 \\ -17678 & -17678 & 17678 & 17678 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -50000 & 0 & 0 & 0 & 50000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Espalhamento de "k3": $[1\ 2\ 7\ 8]$

$$k3 = \begin{bmatrix} 14400 & -19200 & -14400 & 19200 \\ -19200 & 25600 & 19200 & -25600 \\ -14400 & 19200 & 14400 & -19200 \\ 19200 & -25600 & -19200 & 25600 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 7 \\ 8 \end{bmatrix}$$

$$KG := \begin{bmatrix} k1_{1,1}+k2_{1,1}+k3_{1,1} & k1_{1,2}+k2_{1,2}+k3_{1,2} & k1_{1,3} & k1_{1,4} & k2_{1,3} & k2_{1,4} & k3_{1,3} & k3_{1,4} \\ k1_{2,1}+k2_{2,1}+k3_{2,1} & k1_{2,2}+k2_{2,2}+k3_{2,2} & k1_{2,3} & k1_{2,4} & k2_{2,3} & k2_{2,4} & k3_{2,3} & k3_{2,4} \\ k1_{3,1} & k1_{3,2} & k1_{3,3} & k1_{3,4} & 0 & 0 & 0 & 0 \\ k1_{4,1} & k1_{4,2} & k1_{4,3} & k1_{4,4} & 0 & 0 & 0 & 0 \\ k2_{3,1} & k2_{3,2} & 0 & 0 & k2_{3,3} & k2_{3,4} & 0 & 0 \\ k2_{4,1} & k2_{4,2} & 0 & 0 & k2_{4,3} & k2_{4,4} & 0 & 0 \\ k3_{3,1} & k3_{3,2} & 0 & 0 & 0 & 0 & k3_{3,3} & k3_{3,4} \\ k3_{4,1} & k3_{4,2} & 0 & 0 & 0 & 0 & k3_{4,3} & k3_{4,4} \end{bmatrix}$$

$$KG = \begin{bmatrix} 32078 & -1522,3 & -17678 & -17678 & 0 & 0 & -14400 & 19200 \\ -1522,3 & 93278 & -17678 & -17678 & 0 & -50000 & 19200 & -25600 \\ -17678 & -17678 & 17678 & 17678 & 0 & 0 & 0 & 0 \\ -17678 & -17678 & 17678 & 17678 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -50000 & 0 & 0 & 0 & 50000 & 0 & 0 \\ -14400 & 19200 & 0 & 0 & 0 & 0 & 14400 & -19200 \\ 19200 & -25600 & 0 & 0 & 0 & 0 & -19200 & 25600 \end{bmatrix}$$

$$K11 := \begin{bmatrix} KG_{1,1} & KG_{1,2} \\ KG_{2,1} & KG_{2,2} \end{bmatrix} = \begin{bmatrix} 32078 & -1522,3 \\ -1522,3 & 93278 \end{bmatrix} \qquad K12 := \begin{bmatrix} KG_{1,3} & KG_{1,4} & KG_{1,5} & KG_{1,6} & KG_{1,7} & KG_{1,8} \\ KG_{2,3} & KG_{2,4} & KG_{2,5} & KG_{2,6} & KG_{2,7} & KG_{2,8} \end{bmatrix} = \begin{bmatrix} -17678 & -17678 & 0 & 0 & -14400 & 19200 \\ -17678 & -17678 & 0 & -50000 & 19200 & -25600 \end{bmatrix}$$

$$K21 := \begin{bmatrix} KG_{3,1} & KG_{3,2} \\ KG_{4,1} & KG_{4,2} \\ KG_{5,1} & KG_{5,2} \\ KG_{6,1} & KG_{6,2} \\ KG_{7,1} & KG_{7,2} \\ KG_{8,1} & KG_{8,2} \end{bmatrix} = \begin{bmatrix} -17678 & -17678 \\ -17678 & -17678 \\ 0 & 0 \\ 0 & -50000 \\ -14400 & 19200 \\ 19200 & -25600 \end{bmatrix} \qquad K22 := \begin{bmatrix} KG_{3,3} & KG_{3,4} & KG_{3,5} & KG_{3,6} & KG_{3,7} & KG_{3,8} \\ KG_{4,3} & KG_{4,4} & KG_{4,5} & KG_{4,6} & KG_{4,7} & KG_{4,8} \\ KG_{5,3} & KG_{5,4} & KG_{5,5} & KG_{5,6} & KG_{5,7} & KG_{5,8} \\ KG_{6,3} & KG_{6,4} & KG_{6,5} & KG_{6,6} & KG_{6,7} & KG_{6,8} \\ KG_{7,3} & KG_{7,4} & KG_{7,5} & KG_{7,6} & KG_{7,7} & KG_{7,8} \\ KG_{8,3} & KG_{8,4} & KG_{8,5} & KG_{8,6} & KG_{8,7} & KG_{8,8} \end{bmatrix} = \begin{bmatrix} 17678 & 17678 & 0 & 0 & 0 & 0 \\ 17678 & 17678 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 50000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 14400 & -19200 \\ 0 & 0 & 0 & 0 & -19200 & 25600 \end{bmatrix}$$

$$Qk := \begin{bmatrix} -P \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$$Qu := \begin{bmatrix} Q3 \\ Q4 \\ Q5 \\ Q6 \\ Q7 \\ Q8 \end{bmatrix} \qquad Dk := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Vetor de Forças Térmicas na Barra [2]

$$Ft2 := E \cdot A \cdot \alpha \cdot \Delta T \cdot \begin{bmatrix} \lambda x2 \\ \lambda y2 \\ -\lambda x2 \\ -\lambda y2 \end{bmatrix} = \begin{bmatrix} 0 \\ -128,7 \\ 0 \\ 128,7 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 5 \\ 6 \end{bmatrix}$$

Vetor Global de Forças Térmicas:

$$FT := \begin{bmatrix} Ft2_{1} \\ Ft2_{2} \\ 0 \\ 0 \\ Ft2_{3} \\ Ft2_{4} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -128,7 \\ 0 \\ 0 \\ 0 \\ 128,7 \\ 0 \\ 0 \end{bmatrix}$$

$$FT1 := \begin{bmatrix} FT_{1} \\ FT_{2} \end{bmatrix} = \begin{bmatrix} 0 \\ -128,7 \end{bmatrix}$$

$$FT2 := \begin{bmatrix} FT_{3} \\ FT_{4} \\ FT_{5} \\ FT_{6} \\ FT_{7} \\ FT_{8} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 128,7 \\ 0 \\ 0 \end{bmatrix}$$

Solução do Sistema de Equações:

Solução do Sistema de Equações:

Qk - FT1 = K11*Du + K12*Dk (1ª Linha)
Qu - FT2 = K21*Du + K22*Dk (2ª Linha)

Resolvendo a 1ª Linha: Du = K11^-1 * (Qk - FT1 - K12*Dk)

$$Du := K11^{-1} \cdot (Qk - FT1 - K12 \cdot Dk) = \begin{bmatrix} 3,1335 \cdot 10^{-6} \\ 0,0013798 \end{bmatrix}$$

$$D1 := Du_1 = 3,1335 \cdot 10^{-6} \qquad D2 := Du_2 = 0,0013798 \qquad D5 := 0 \qquad D6 := 0$$

Resolvendo a 2ª Linha: Qu = K21*Du + K22*Dk + FT2

$$Qu := K21 \cdot Du + K22 \cdot Dk + FT2 = \begin{bmatrix} -24,4471 \\ -24,4471 \\ 0 \\ 59,7099 \\ 26,4471 \\ -35,2628 \end{bmatrix}$$

Reações de apoio nos graus de liberdade 3, 4, 5, 6, 7 e 8 respectivamente

Esforços Axiais nas Barras:

Sem carga térmica:

$$q_F = \frac{AE}{L}[-\lambda_x \quad -\lambda_y \quad \lambda_x \quad \lambda_y] \begin{Bmatrix} D_{Nx} \\ D_{Ny} \\ D_{Fx} \\ D_{Fy} \end{Bmatrix}$$

Com carga térmica:

$$N := \frac{A \cdot E}{L} \cdot \left(\left([-\lambda x2 \quad -\lambda y2 \quad \lambda x2 \quad \lambda y2] \cdot \begin{bmatrix} DNx \\ DNy \\ DFx \\ DFy \end{bmatrix} \right)_1 \right) - E \cdot A \cdot \alpha \cdot \Delta T$$

$$Ft2 := E \cdot A \cdot \alpha \cdot \Delta T = 128,7$$

Na Barra 2:

[1 2 5 6]

$$N2 := \frac{E \cdot A}{L2} \cdot \left(\left([-\lambda x2 \quad -\lambda y2 \quad \lambda x2 \quad \lambda y2] \cdot \begin{bmatrix} D1 \\ D2 \\ D5 \\ D6 \end{bmatrix} \right)_1 \right) - E \cdot A \cdot \alpha \cdot \Delta T = -59,7099$$

$$N2 := \frac{E \cdot A}{L2} \cdot \left(\left([-\lambda x2 \quad -\lambda y2 \quad \lambda x2 \quad \lambda y2] \cdot \begin{bmatrix} D1 \\ D2 \\ D5 \\ D6 \end{bmatrix} \right)_1 \right) = 68,9901$$