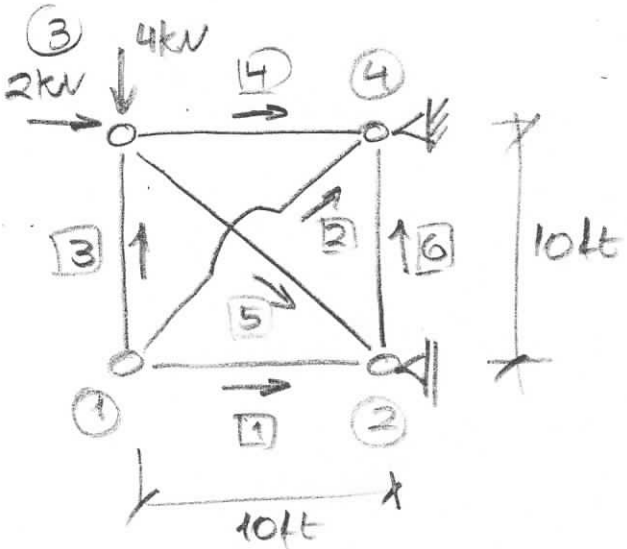
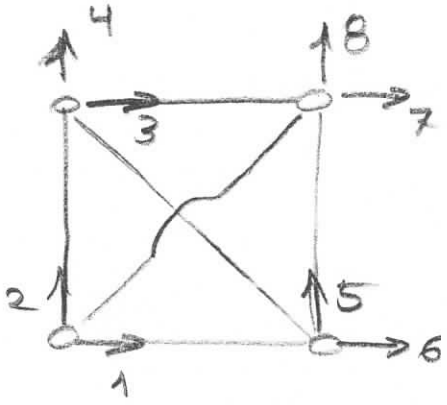
TE2-8.4





BARRA 1 $\lambda x = \cos 0° = 1$ $\lambda y = \cos 90° = 0$ $L = 10\,ft$

$$[k1] = \frac{AE}{10}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = AE\begin{bmatrix} 1/10 & 0 & -1/10 & 0 \\ 0 & 0 & 0 & 0 \\ -1/10 & 0 & 1/10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{matrix}1\\2\\6\\5\end{matrix}$$

(columnas: 1, 2, 6, 5)

BARRA 2 $\lambda x = \cos 45° = \sqrt{2}/2$ $\lambda y = \cos 45° = \sqrt{2}/2$ $L = 10\sqrt{2}\,ft$

$$[k2] = \frac{AE}{10\sqrt{2}}\begin{bmatrix} 1/2 & 1/2 & -1/2 & -1/2 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \end{bmatrix} = AE\begin{bmatrix} \sqrt{2}/40 & \sqrt{2}/40 & -\sqrt{2}/40 & -\sqrt{2}/40 \\ & \sqrt{2}/40 & -\sqrt{2}/40 & -\sqrt{2}/40 \\ & & \sqrt{2}/40 & \sqrt{2}/40 \\ \text{SIM} & & & \sqrt{2}/40 \end{bmatrix}\begin{matrix}1\\2\\7\\8\end{matrix}$$

(columnas: 1, 2, 7, 8)

BARRA 3 $\lambda x = \cos 90° = 0$ $\lambda y = \cos 0° = 1$ $L = 10\,ft$

$$[k3] = \frac{AE}{10}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} = AE\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/10 & 0 & -1/10 \\ 0 & 0 & 0 & 0 \\ 0 & -1/10 & 0 & 1/10 \end{bmatrix}\begin{matrix}1\\2\\3\\4\end{matrix}$$

(columnas: 1, 2, 3, 4)

BARRA (4) Igual à barra (1) porém com outros índices:

$$[K4] = AE \begin{bmatrix} 3 & 4 & 7 & 8 \\ 1/10 & 0 & -1/10 & 0 \\ 0 & 0 & 0 & 0 \\ -1/10 & 0 & 1/10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 3 \\ 4 \\ 7 \\ 8 \end{matrix}$$

BARRA (5) $\lambda x = \cos -45^\circ = \sqrt{2}/2$ $\lambda y = \cos -135^\circ = -\sqrt{2}/2$ $L = 10\sqrt{2}$ ft

$$[K5] = \frac{AE}{10\sqrt{2}} \begin{bmatrix} 1/2 & -1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & -1/2 & 1/2 \end{bmatrix} = AE \begin{bmatrix} 3 & 4 & 6 & 5 \\ \sqrt{2}/40 & -\sqrt{2}/40 & -\sqrt{2}/40 & \sqrt{2}/40 \\ & \sqrt{2}/40 & \sqrt{2}/40 & -\sqrt{2}/40 \\ & & \sqrt{2}/40 & -\sqrt{2}/40 \\ \text{SIM} & & & \sqrt{2}/40 \end{bmatrix} \begin{matrix} 3 \\ 4 \\ 6 \\ 5 \end{matrix}$$

BARRA (6) Igual à barra (3) porém com outros índices:

$$[K6] = AE \begin{bmatrix} 6 & 5 & 7 & 8 \\ 0 & 0 & 0 & 0 \\ 0 & 1/10 & 0 & -1/10 \\ 0 & 0 & 0 & 0 \\ 0 & -1/10 & 0 & 1/10 \end{bmatrix} \begin{matrix} 6 \\ 5 \\ 7 \\ 8 \end{matrix}$$

- MONTAGEM DA MATRIZ DE RIGIDEZ GLOBAL:

$K_{11} = K1_{11} + K2_{11} + K3_{11} = 1/10 + \sqrt{2}/40 + 0 = 0{,}1354$

$K_{12} = K1_{12} + K2_{12} + K3_{12} = 0 + \sqrt{2}/40 + 0 = 0{,}03536$

$K_{13} = 0$

$K_{14} = 0$

$K_{15} = 0$

$K_{16} = -0{,}1$

$K_{17} = -\sqrt{2}/40 = -0{,}03536$

$K_{18} = -\sqrt{2}/40 = -0{,}03536$

$k_{22} = k1_{22} + k2_{22} + k3_{22} = 0 + \frac{\sqrt{2}}{40} + \frac{1}{10} = 0,1354$

$k_{23} = 0$

$k_{24} = -0,1$

$k_{25} = 0$

$k_{26} = 0$

$k_{27} = -\sqrt{2}/40 = -0,03536$

$k_{28} = -\sqrt{2}/40 = -0,03536$

$k_{33} = k3_{33} + k4_{33} + k5_{33} = 0 + \frac{1}{10} + \frac{\sqrt{2}}{40} = 0,1354$

$k_{34} = k3_{34} + k4_{34} + k5_{34} = 0 + 0 - \sqrt{2}/40 = -0,03536$

$k_{35} = \sqrt{2}/40 = 0,03536$

$k_{36} = -\sqrt{2}/40 = -0,03536$

$k_{37} = -0,1$

$k_{38} = 0$

$k_{44} = k3_{44} + k4_{44} + k5_{44} = \frac{1}{10} + 0 + \frac{\sqrt{2}}{10} = 0,1354$

$k_{45} = -\sqrt{2}/40 = -0,03536$

$k_{46} = \sqrt{2}/40 = 0,03536$

$k_{47} = 0$

$k_{48} = 0$

$k_{55} = k1_{55} + k5_{55} + k6_{55} = 0 + \frac{\sqrt{2}}{40} + \frac{1}{10} = 0,1354$

$k_{56} = k1_{56} + k5_{56} + k6_{56} = 0 - \frac{\sqrt{2}}{40} + 0 = -0,03536$

$k_{57} = 0$

$k_{58} = -0,1$

$$K_{66} = K1_{66} + K5_{66} + K6_{66} = 1/10 + \sqrt{2}/40 + 0 = 0,1354$$

$$K_{67} = 0$$

$$K_{68} = 0$$

$$K_{77} = K2_{77} + K4_{77} + K6_{77} = \frac{\sqrt{2}}{40} + \frac{1}{10} + 0 = 0,1354$$

$$K_{78} = K2_{78} + K4_{78} + K6_{78} = \frac{\sqrt{2}}{40} + 0 + 0 = 0,03536$$

$$K_{88} = K2_{88} + K4_{88} + K6_{88} = \frac{\sqrt{2}}{40} + 0 + \frac{1}{10} = 0,1354$$

- VETOR DE CARGAS / REAÇÕES DE APOIO:

$$\{Q\} = \begin{cases} 0 \\ 0 \\ 2 \\ -4 \\ 0 \\ Q_6 \\ Q_7 \\ Q_8 \end{cases} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix}$$

1 a 5: CARGAS APLICADAS NOS NÓS

6 a 8: REAÇÕES DE APOIO (FORÇAS INCÓGNITAS)

- VETOR DE DESLOCAMENTOS:

$$\{D\} = \begin{cases} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ 0 \\ 0 \\ 0 \end{cases}$$

D_1 a D_5: DESLOCAMENTOS NODAIS DESCONHECIDOS

0 a 0 a 0: DESLOCAMENTOS NODAIS CONHECIDOS (VÍNCULOS EXTERNOS)

SISTEMA LINEAR:

$$\{Q\} = [K] \cdot \{D\}$$

$$\begin{Bmatrix} 0 \\ 0 \\ 2 \\ -4 \\ 0 \\ \hline Q_6 \\ Q_7 \\ Q_8 \end{Bmatrix} = \left[\begin{array}{ccccc|ccc} 0,1354 & 0,03536 & 0 & 0 & 0 & -0,1 & -0,03536 & -0,03536 \\ 0,03536 & 0,1354 & 0 & -0,1 & 0 & 0 & -0,03536 & -0,03536 \\ 0 & 0 & 0,1354 & -0,03536 & 0,03536 & -0,03536 & -0,1 & 0 \\ 0 & -0,1 & -0,03536 & 0,1354 & -0,03536 & 0,03536 & 0 & 0 \\ 0 & 0 & 0,03536 & -0,03536 & 0,1354 & -0,03536 & 0 & -0,1 \\ \hline -0,1 & 0 & -0,03536 & 0,03536 & -0,03536 & 0,1354 & 0 & 0 \\ -0,03536 & -0,03536 & -0,1 & 0 & 0 & 0 & 0,1354 & 0,03536 \\ -0,03536 & -0,03536 & 0 & 0 & -0,1 & 0 & 0,03536 & 0,1354 \end{array}\right] \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

- Cálculo dos Deslocamentos:

$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{Bmatrix} = \frac{1}{EA} \begin{bmatrix} 0,1354 & 0,03536 & 0 & 0 & 0 \\ 0,03536 & 0,1354 & 0 & -0,1 & 0 \\ 0 & 0 & 0,1354 & -0,03536 & 0,03536 \\ 0 & -0,1 & -0,03536 & 0,1354 & -0,03536 \\ 0 & 0 & 0,03536 & -0,03536 & 0,1354 \end{bmatrix}^{-1} \begin{Bmatrix} 0 \\ 0 \\ 2 \\ -4 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{Bmatrix} = \frac{1}{EA} \begin{Bmatrix} 17,89 \\ -68,49 \\ -2,043 \\ -86,42 \\ -22,03 \end{Bmatrix}$$

//

• Cálculo das Reações de Apoio:

$$\begin{Bmatrix} Q_6 \\ Q_7 \\ Q_8 \end{Bmatrix} = EA \begin{bmatrix} -0,1 & 0 & -0,03536 & 0,03536 & -0,03536 \\ -0,03536 & -0,03536 & -0,1 & 0 & 0 \\ -0,03536 & -0,03536 & 0 & 0 & -0,1 \end{bmatrix} \frac{1}{EA} \begin{Bmatrix} 17,89 \\ -68,49 \\ -2,043 \\ -86,42 \\ -22,03 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} Q_6 \\ Q_7 \\ Q_8 \end{Bmatrix} = \begin{Bmatrix} -3,993 \\ 1,994 \\ 3,993 \end{Bmatrix} KN$$

//

• Cálculo de Esforço Interno na Barra (2):

$$q_{(2)} = \frac{AE}{L_{(2)}} \left[-\lambda x_{(2)} \; -\lambda y_{(2)} \; \lambda x_{(2)} \; \lambda y_{(2)} \right] \frac{1}{EA} \begin{Bmatrix} D_1 \\ D_2 \\ D_7 \\ D_8 \end{Bmatrix}$$

$$q_{(2)} = \frac{1}{10\sqrt{2}} \left[-\sqrt{2}/2 \; -\sqrt{2}/2 \; \sqrt{2}/2 \; \sqrt{2}/2 \right] \begin{Bmatrix} 17,89 \\ -68,49 \\ 0 \\ 0 \end{Bmatrix}$$

$$q_{(2)} = \frac{1}{10\sqrt{2}} (-12,65 + 48,43) = +2,53 \; KN \; // \; \text{TRAÇÃO}$$